MATH 101.15 (Term 171)

Quiz 1 (Sects. 2.2, 2.3 & 2.4) Duration: 20min

Name: ID number:

1.)(3pts)Evaluate the limit if it exists

$$a.) \lim_{x\to 0} (\sin x)^2 \cos(\frac{\pi}{x}), \quad \lim_{x\to 0^+} [[\cos(x+\frac{\pi}{2})]] + 2x - 1.$$

2.)(4pts)Find all vertical asymptote to the curve of the function $f(x) = \frac{x^2+x-2}{x^2-1}$.

3.)(3pts)Use the ε-δ definition of limit to prove that $\lim_{x\to -1} 3x+1 = -2$.

1) a) $-1 \le \cos\frac{\pi}{x} \le 1, \quad x \ne 0$

$\Rightarrow -(\sin x)^2 \le (\sin x)^2 \cos\frac{\pi}{x} \le (\sin x)^2$

$\lim_{x\to 0} (\sin x)^2 = 0$; $\lim_{x\to 0} -(\sin x)^2 = 0$

By the squeezing theorem,

$\lim_{x\to 0} (\sin x)^2 \cos\frac{\pi}{x} = 0$

b) when $x \ge 0$, $x+\frac{\pi}{2} \ge \pi/2$

$\lim_{x\to 0^+} [[\cos(x+\frac{\pi}{2})]] = -1$,

and $\lim_{x\to 0^+} [[\cos(x+\frac{\pi}{2})]] + 2x - 1 = -2$

2) $f(x) = \frac{x^2+x-2}{x^2-1}$

$x^2-1=0$, $(x-1)(x+1)=0$

$x=-1$, $x=1$.

$\lim_{x\to -1^+} f(x) = \frac{-2}{0^-} = +\infty$.

$\boxed{x=-1 \text{ is a Vertical Asymptote}}$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{(x-1)(x+2)}{(x-1)(x+1)} = \frac{3}{2}$.

$x=1$ is not a Vertical Asymptote

3) let $\varepsilon > 0$ be arbitrary.

Find $\delta > 0$ such that

$|x+1| < \delta \Rightarrow |3x+1+2| < \varepsilon$

$\Updownarrow$

$3|x+1| < \varepsilon$

$\Leftrightarrow |x+1| < \frac{\varepsilon}{3}$

We can choose any δ such that

$\boxed{0 < \delta \le \varepsilon/3}$