

Name: _____

ID number: _____

1.) (3pts) Evaluate $\lim_{z \rightarrow -2} \frac{\frac{z}{z^2+1} + \frac{2}{5}}{z+2}$

2.) (4pts) Find all horizontal asymptotes to $f(x) = \begin{cases} \frac{\sqrt{x^2-x}}{2x}, & x < 0 \\ \frac{e^x-1}{e^{2x}+1}, & x \geq 0. \end{cases}$

3.) (3pts) Find values of a to make $g(x) = \begin{cases} \lfloor x-2 \rfloor, & x < 1 \\ -x^2+ax+3, & x \geq 1 \end{cases}$ continuous at $x=1$.

1.) $\lim_{z \rightarrow -2} \frac{\frac{z}{z^2+1} + \frac{2}{5}}{z+2} = \lim_{z \rightarrow -2} \frac{(z+2)(2z+1)}{5(z^2+1)(z+2)}$

$= \lim_{z \rightarrow -2} \frac{2z+1}{5(z^2+1)}$

$= -\frac{3}{25}$

2.) $\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{e^x-1}{e^{2x}+1}$
 $= \lim_{x \rightarrow +\infty} \frac{e^x(1-e^{-x})}{e^{2x}(1+e^{-2x})}$

$= \lim_{x \rightarrow +\infty} \frac{1-e^{-x}}{e^x(1+e^{-x})}$

$= 0$

$y=0$ is HA

$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2-x}}{2x}$
 $= \lim_{x \rightarrow -\infty} \frac{-x\sqrt{1-\frac{1}{x}}}{2x}$
 $= \lim_{x \rightarrow -\infty} -\frac{\sqrt{1-\frac{1}{x}}}{2} = -\frac{1}{2}$

$y = -\frac{1}{2}$ is a HA

3.) $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} f(x) = f(1)$

$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \lfloor x-2 \rfloor = -2$

$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (-x^2+ax+3) = a+2$

$\Rightarrow a+2 = -2$

$a = -4$