MATH 101.15 (Term 171)

Quiz 3 (Sects. 3.2, 3.3 & 3.4) Duration: 20min

Name: ID number:

1.)(3pts) If $f(t)=\frac{1}{\sqrt{t}}+ t^2(t+1)$, find $f''(1)$.

2.)(4pts)Find all horizontal tangent to the curve of $y=\frac{\cos x-\sin x}{\cos x+\sin x}$, $-\frac{\pi}{4}<x<\frac{3\pi}{4}$.

3.)(3pts)Find an equation of the tangent line to the curve of $f(x)=\sin^2(\frac{\pi}{3}e^{x^3-1})$ at $x=1$.

1.) $f'(t)=-\frac{1}{2}t^{-\frac{3}{2}}+2t(t+1)+t^2$

$=-\frac{1}{2}t^{-\frac{3}{2}}+3t^2+2t$

$f''(t)=-\frac{1}{2}\left(-\frac{3}{2}\right)t^{-\frac{5}{2}}+6t+2$

$f''(t)=\frac{3}{4}t^{-\frac{5}{2}}+6t+2$

$\Rightarrow f''(1)=\frac{3}{4}+6+2=\frac{35}{4}$

2.) $f(x)=\frac{\cos x-\sin x}{\cos x+\sin x}$

$f'(x)=\frac{(-\sin x-\cos x)(\cos x+\sin x)-(-\sin x+\cos x)(\cos x-\sin x)}{(\cos x+\sin x)^2}$

$=\frac{-(\cos x+\sin x)^2-(\cos x-\sin x)^2}{(\cos x+\sin x)^2}$

$=\frac{-2(\cos^2 x+\sin^2 x)}{(\cos x+\sin x)^2}=\frac{-2}{(\cos x+\sin x)^2}$

$f'(x)\neq 0$ for every $x$

There is no horizontal tangent

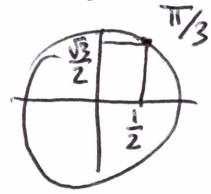
3.) $y-f(1)=f'(1)(x-1)$

$f'(x)=2\sin\left(\frac{\pi}{3}e^{x^3-1}\right)\left[\sin\left(\frac{\pi}{3}e^{x^3-1}\right)\right]'$

$\left[\sin\left(\frac{\pi}{3}e^{x^3-1}\right)\right]'=\frac{\pi}{3}(3x^2)e^{x^3-1}\cos\left(\frac{\pi}{3}e^{x^3-1}\right)$

$f'(x)=2\pi x^2e^{x^3-1}\sin\left(\frac{\pi}{3}e^{x^3-1}\right)\cos\left(\frac{\pi}{3}e^{x^3-1}\right)$

$f'(1)=2\pi\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)$




$f'(1)=2\pi\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi\sqrt{3}}{2}$

$f(1)=\sin^2\left(\frac{\pi}{3}\right)=\left(\frac{\sqrt{3}}{2}\right)^2=\frac{3}{4}$

$y-\frac{3}{4}=\frac{\pi\sqrt{3}}{2}(x-1)$

$\boxed{y=\frac{\pi\sqrt{3}}{2}x+\frac{3}{4}-\frac{\pi\sqrt{3}}{2}}$