

MATH 101.15 (Term 171)

Quiz 4 (Sects. 3.5, 3.6 & 3.7)

Duration: 20min

Name:

ID number:

1.) (3pts) Find the slope of the tangent line to: $\tan^{-1}\left(\frac{xy}{4}\right) = \frac{\pi}{32}(x^2 + y^2)$ at (2, 2).

2.) (3pts) Find $f'(0)$ if $f(x) = \frac{(x-2)^4(x^2+3)^3}{(x-1)^2 \cos x}$.

3.) (4pts) A particle moves with the position function $s(t) = \sin\left(\frac{\pi}{2}t\right) + \cos\left(\frac{\pi}{2}t\right)$ (t in seconds and s in meters). Find the total distance traveled by the particle during the time $t \in [0, 1]$.

1) $\tan^{-1}\left(\frac{xy}{4}\right) = \frac{\pi}{32}(x^2 + y^2)$

We differentiate with respect to x

$$\frac{1}{4}(y + x \frac{dy}{dx}) \frac{1}{1 + \left(\frac{xy}{4}\right)^2} = \frac{\pi}{32}(2x + 2y \frac{dy}{dx})$$

let $m = \frac{dy}{dx} \Big|_{x=2, y=2}$

$$\frac{1}{4}(2 + 2m) \frac{1}{1 + 1} = \frac{\pi}{32}(4 + 4m)$$

$$\frac{1}{4} + \frac{m}{4} = \frac{\pi}{8} + \frac{\pi}{8}m$$

$$m = \frac{\frac{1}{4} - \frac{\pi}{8}}{\frac{\pi}{8} - \frac{1}{4}} = -1$$

2) $f(x) = \frac{(x-2)^4(x^2+3)^3}{(x-1)^2 \cos x}$

$$\ln f(x) = 4 \ln|x-2| + 3 \ln(x^2+3) - 2 \ln|x-1| - \ln \cos x$$

We differentiate $\ln f(x)$,

$$\frac{f'(x)}{f(x)} = \frac{4}{x-2} + \frac{6x}{x^2+3} - \frac{2}{x-1} + \tan x$$


$$\frac{f'(0)}{f(0)} = -2 + 2 = 0$$

$$f(0) = 2^4 \cdot 3^3 \neq 0 \Rightarrow f'(0) = 0$$

3) $s(t) = \sin \frac{\pi t}{2} + \cos \frac{\pi t}{2}$

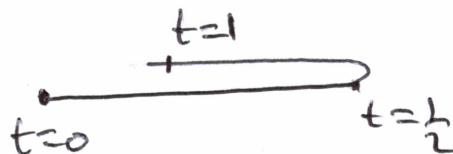
$v(t) = \frac{\pi}{2} \cos \frac{\pi t}{2} - \frac{\pi}{2} \sin \frac{\pi t}{2}$: velocity

$v(t) = 0 \Leftrightarrow \frac{\pi}{2}(\cos \frac{\pi t}{2} - \sin \frac{\pi t}{2}) = 0$

 $\tan \frac{\pi t}{2} = 1$
 $\frac{\pi t}{2} = \frac{\pi}{4} + k\pi$

$t = \frac{1}{2} + 2k$

t	0	$\frac{1}{2}$	1
$v(t)$	+	0	-



distance = $|s(\frac{1}{2}) - s(0)| + |s(1) - s(\frac{1}{2})|$

$s(0) = 1, s(\frac{1}{2}) = \sqrt{2}, s(1) = 1$

\Rightarrow distance = $\sqrt{2} - 1 + |1 - \sqrt{2}|$
 $= 2(\sqrt{2} - 1)$