MATH 101.15 (Term 171)

Quiz 5 (Sects. 4.1, 4.2 & 4.3) Duration: 20min

Name: ID number:

1.)(3pts) Find the absolute maximum and minimum values of the function $f(x) = \frac{\sin x}{2+\cos x}$ on $[0, 2\pi]$.

2.)(3pts) Show that the equation $x^5 + x + 1 = 0$ has exactly one solution.

3.)(4pts) Study the concavity and find inflection points of the function $g(x) = \ln(x^2+1)$.

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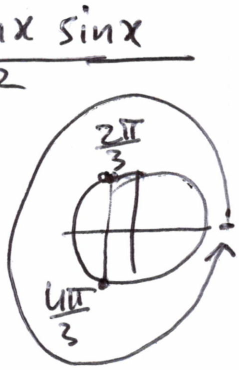
1) $f'(x) = \dfrac{\cos x(2+\cos x) + \sin x \sin x}{(2+\cos x)^2}$

$= \dfrac{2\cos x + 1}{(2+\cos x)^2}$



$f'(x) = 0 \Rightarrow \cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \quad \frac{4\pi}{3}$

Two critical numbers

$f\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{3}, \quad f\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

Now, we calculate $f(0)$ and $f(2\pi)$

$f(0) = 0, \quad f(2\pi) = 0$

$\Rightarrow f\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{3}$ Absolute maximum value

$f\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{3}$ Absolute minimum value

2) Let $f(x) = x^5 + x + 1$

$f(0) = 1 > 0, \quad f(-1) = -1 < 0$

$\xRightarrow{\text{IVT}}$ there exists $c \in (-1, 0)$ such that $f(c) = 0$.

Assume there is another point d such that $f(d) = 0$.

$\xRightarrow{\text{Rolle's theorem}}$ there exists $\alpha \in (c, d)$ such that $f'(\alpha) = 0$.

But, $f'(x) = 5x^4 + 1 > 0$

Impossible to have $f'(\alpha) = 0$

$\Rightarrow$ we cannot have two solutions.

The equation $f(x) = 0$ has only one solution

3) $g(x) = \ln(x^2+1)$

$g'(x) = \dfrac{2x}{x^2+1}$

$g''(x) = \dfrac{2(1-x^2)}{(x^2+1)^2}$

x	$-\infty$		-1		1		$+\infty$
$g''(x)$		$-$	0	$+$	0	$-$	
		Concave downward		Concave upward		Concave downward	

$A(-1, \ln 2)$ and $B(1, \ln 2)$ are the inflections points