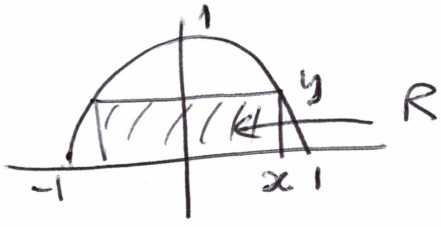
MATH 101.15 (Term 171)
Quiz 6 (Sects. 4.7, 4.8 & 4.9) **Duration: 20min**

Name: **ID number:**

1.)(4pts) Find the area of the largest rectangle that can be inscribed in the curve of equation $y = 1 - x^2$ on $[-1, 1]$.
2.)(3pts) Use Newton's method to find the second approximation x_2 of the root of the equation $x \sin^{-1} x = 1$ if $x_1 = \frac{1}{2}$.
3.)(3pts) Find $f(x)$ if $f''(x) = \frac{1}{(1+x)^2} + \cosh x$, $f(0) = -1$ and $f(1) = 0$.

1.)



Area of $R = 2xy$, $x, y \geq 0$

$y = 1 - x^2$

let $f(x) = 2x(1-x^2)$

$f'(x) = 2[1 - x^2 - 2x^2]$

$= 2[1 - 3x^2]$

$f'(x) = 0 \Rightarrow x^2 = \frac{1}{3}$

$x = \frac{\sqrt{3}}{3}$

x	0		$\sqrt{3}/3$		1
$f'(x)$		+	0	−	
$f(x)$		↗		↘	

$x = \frac{\sqrt{3}}{3}$, $y = \frac{2}{3}$

$$\boxed{f\left(\frac{\sqrt{3}}{3}\right) = \frac{4\sqrt{3}}{9}}$$

2.) $f(x) = x \sin^{-1} x - 1$

$f'(x) = \sin^{-1} x + \frac{x}{\sqrt{1-x^2}}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$,

$f\left(\frac{1}{2}\right) = \frac{1}{2}\frac{\pi}{6} - 1$

$f'\left(\frac{1}{2}\right) = \frac{\pi}{6} + \frac{\sqrt{3}}{2}$

$$\Rightarrow \boxed{x_2 = \frac{1}{2} - \frac{\frac{\pi}{12} - 1}{\frac{\pi}{6} + \frac{\sqrt{3}}{2}}}$$

3.) $f'(x) = -\frac{1}{x+1} + \sinh x + C_1$

$f(x) = -\ln|x+1| + \cosh x + C_1 x + C_2$

$f(0) = -1 \Rightarrow 1 + C_2 = -1 \Rightarrow \boxed{C_2 = -2}$

$f(1) = 0 \Rightarrow -2\ln 2 + \cosh 1 + C_1 - 2 = 0$

$$\boxed{C_1 = 2\ln 2 - \cosh 1 + 2}$$