King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 101 Section 33 Quiz IV (A) (Term 171)

Name : ..... KEY ..... ID #..... Serial #: .....

1. $\lim_{x\to\infty}(1+2x)^{\frac{1}{2\ln x}} =$

Take ln of $f(x)=(1+2x)^{\frac{1}{2\ln x}}$ and then find the limit.

$\ln(1+2x) = \lim_{x\to\infty} \frac{\ln(1+2x)}{2\ln x}$

LHS $= \lim_{x\to\infty} \frac{\frac{2}{1+2x}}{\frac{2}{x}} = \lim_{x\to\infty} \frac{x}{1+2x} = \frac{1}{2}$

a) e^2
b) $\sqrt{e}$
c) $\frac{1}{\sqrt{e}}$
d) e
e) 1

2. The absolute maximum value of

$$f(x) = 2\sin(2x) + \cos(4x)$$

on the interval $\left[0, \frac{\pi}{6}\right]$ is equal to

$f'(x) = 4\cos(2x) - 4\sin(4x) = 0$

$\Rightarrow \cos(2x) - 2\sin(2x)\cos(2x) = 0$

$\Rightarrow \cos 2x\,(1-2\sin 2x) = 0$

$\Rightarrow \cos 2x = 0$ or $\sin 2x = \frac{1}{2}$

$\Rightarrow 2x = \frac{\pi}{2}$ or $2x = \frac{\pi}{6}$

$\Rightarrow x = \frac{\pi}{4}$ X $\Rightarrow x = \frac{\pi}{12}$ ✓

$f(0) = 1$

$f\left(\frac{\pi}{12}\right) = 2\left(\frac{1}{2}\right) + \frac{1}{2} = 3/2$ ✓

$f\left(\frac{\pi}{6}\right) = \frac{2\sqrt{3}}{2} - \frac{1}{2} = \sqrt{3} - \frac{1}{2}$

a) $\frac{3}{2}$
b) 1
c) 3
d) $\frac{\sqrt{3}}{2}$
e) $\frac{1+\sqrt{3}}{2}$

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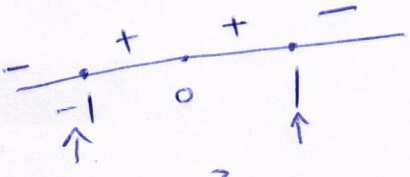
3. The function $f(x) = 5x^3 - 3x^5$ has

a) 2 local extrema and 2 points of inflection
b) 2 critical points and 3 points of inflection
c) 3 critical points and no points of inflection
d) 2 local extrema and 3 points of inflection
e) 2 local extrema and one point of inflection

$f'(x) = 15x^2 - 15x^4$

$= 15x^2(1-x^2) = 15x^2(1-x)(1+x)$



$f''(x) = 30x - 60x^3$

$= 30x(1-2x^2) = 30x(1-\sqrt{2}x)(1+\sqrt{2}x)$

4. If c is a number which satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \sin^{-1}\left(\frac{x}{2}\right)$ on the interval $[0, 2]$, then $\pi^2 c^2 + 16 =$

a) π^2
b) 0
c) $2\pi^2$
d) 4
e) $4\pi^2$

$f'(x) = \dfrac{1/2}{\sqrt{1-\frac{x^2}{4}}}$

MVT $\Rightarrow \dfrac{f(b)-f(a)}{b-a} = f'(c)$

$\Rightarrow \dfrac{f(2)-f(0)}{2} = \dfrac{1}{2\sqrt{1-\frac{c^2}{4}}}$

$\Rightarrow \frac{\pi}{2}\sqrt{1-\frac{c^2}{4}} = 1 \Rightarrow \sqrt{1-\frac{c^2}{4}} = \frac{2}{\pi}$

$\Rightarrow 1-\frac{c^2}{4} = \frac{4}{\pi^2} \Rightarrow \frac{-c^2}{4} = \frac{4}{\pi^2} - 1$

$\Rightarrow c^2 = 4 - \frac{16}{\pi^2}$

$\Rightarrow \pi^2 c^2 + 16 = 4\pi^2 - 16 + 16 = 4\pi^2$