King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 202
EXAM I
2017-2018 (171)

Thursday, Oct. 19, 2017 **Allowed Time: 2 Hours**

Name: _____

ID Number: _____ Serial Number: _____

Section Number: _____ Instructor's Name: _____

Instructions:

1. Write neatly and legibly. You may lose points for messy work.

2. **Show all your work.** No points for answers without justification.

3. **Calculators and Mobiles are not allowed**.

4. Make sure that you have 9 different problems (8 pages + cover page).

Problem No.	Points	Maximum Points
1		11
2		7
3		7
4		21
5		13
6		7
7		6
8		14
9		14
Total:		100

Q1. Consider the differential equation $\mathbf{y'' + 2y' + y = 0}$.

(a) (**7 points**) Verify that $\mathbf{y = c_1 e^{-x} + c_2 x e^{-x}}$ is a two parameters family of solutions of the given differential equation.

(b) (**4 points**) Find the member of the family of solutions $\mathbf{y = c_1 e^{-x} + c_2 x e^{-x}}$ that satisfies the boundary conditions

$$\mathbf{y(0) = 1,\ y(1) = 3}.$$

Q2. (**7 points**) Find the **values** of $\boldsymbol{b}$ for which the IVP

$$\frac{dy}{dx} = \frac{\sqrt{y - 6x}}{x^2 + 1}, y(5) = b$$

has a unique solution using theorem **"Existence of a Unique Solution"**.

Q3. (**7 points**) Use an appropriate substitution to reduce the differential equation

$$\frac{dy}{dx} = \frac{y^2 + x^2}{xy}$$

to a separable equation. (Note: **Do not solve** the new differential equation)

Q4. Consider the ordinary differential equation

$$\frac{dy}{dx} = \frac{y^2-4}{x^2-1}.$$

(a) (**13 points**) Find the explicit solution of the given differential equation.

(b) (**2 points**) Guess a solution of the IVP

$$\frac{dy}{dx} = \frac{y^2-4}{x^2-1}, y(2) = -2.$$

(c) (**6 points**) Determine the interval of validity of the solution of the IVP given in part (b)

Q5. Consider the differential equation

$$(\boldsymbol{x\,sin\,y + cosy})\frac{\boldsymbol{dy}}{\boldsymbol{dx}} + (\boldsymbol{x + y})\boldsymbol{siny} = \mathbf{0}.$$

(a) (**6 points**) Find an integrating factor that makes the differential equation exact.

(b) (**7 points**) Solve the differential equation using the integrating factor obtained in (a).

Q6. (**7 points**) Use an appropriate substitution to reduce the differential equation

$$\frac{dy}{dx} = y(xy^5 - 1)$$

to a linear equation. (Note: **Do not solve** the new differential equation)

Q7. (**6 points**) Use an appropriate substitution to reduce the differential equation

$$\frac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$$

to a separable equation. (Note: **Do not solve** the new differential equation)

Q8. (**14 points**) A thermometer reading **70** Fahrenheit is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer records that the thermometer reads **110** Fahrenheit after $\frac{1}{2}$ minute and **145** Fahrenheit after **1** minute. What is the constant preheating temperature of the oven?

Q9. (**14 points**) Find the **continuous** solution of the IVP

$$(\mathbf{1}+\boldsymbol{x}^{\mathbf{2}})\frac{\boldsymbol{dy}}{\boldsymbol{dx}}+\mathbf{2}\boldsymbol{xy}=\boldsymbol{f}(\boldsymbol{x}),\ \boldsymbol{y}(\mathbf{0})=\mathbf{0},$$

where

$$\boldsymbol{f}(\boldsymbol{x})=\begin{cases}\boldsymbol{x}, & \boldsymbol{x}<\mathbf{1}\\ -\boldsymbol{x}, & \boldsymbol{x}\geq\mathbf{1}\end{cases}.$$