King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics
MATH 202
EXAM II
2017-2018 (171)

Thursday, Nov. 23, 2017 **Allowed Time: 2 Hours**

Name: ____

ID Number: ________________________ **Serial Number:** ________________

Section Number: ______________ **Instructor's Name:** __________________

Instructions:

1. Write neatly and legibly. You may lose points for messy work.

2. **Show all your work.** No points for answers without justification.

3. **Calculators and Mobiles are not allowed**.

4. Make sure that you have 8 different problems (10 pages).

Problem No.	Points	Maximum Points
1		7
2		12
3		6
4		13
5		15
6		17
7		16
8		14
Total:		100

Q1. (**7 points**) Use the **existence and uniqueness theorem** to find the **largest interval** so that the given initial value problem has a unique solution

$$(x-2)y'' + \ln(x+2)\,y = x, \quad y(0)=0, \quad y'(0)=1.$$

Q2. Let $y_1 = \cos(\ln x)$ and $y_2 = \sin(\ln x)$ be both solutions of the differential equation

$$x^2 y'' + x y' + y = 0.$$

(a) (**7 points**) Use the Wronskian to verify that y_1 and y_2 form a fundamental set of solutions of the given differential equation on the interval $(0, \infty)$.

(b) (**5 points**) Use part (a) to verify that $y = c_1 \cos(\ln x) + c_2 \sin(\ln x) + 1$ is the general solution of the nonhomogeneous differential equation

$$x^2 y'' + x y' + y = 1.$$

Q3. (**6 points**) Given that $y_{p_1} = -2-2e^{x}(x+3)$ and $y_{p_2} = \frac{3}{2} - e^{-x}(x+1)$ are, respectively, particular solutions of the differential equations $y'''-2y = 4+2xe^{x}$ and $y'''-2y = -3+3xe^{-x}$, find a particular solution of the differential equation

$$y'''-2y = \frac{1}{2} + x\cosh x.$$

Q4. (**13 points**) Given that $y_1 = xe^{5x}$ is a solution of

$$y^{(4)} - 12y^{(3)} + 47y'' - 70y' + 50y = 0,$$

find the general solution of the given differential equation.

Q5. (**15 points**) Solve the differential equation

$$y'' - 2y' - 3y = 4e^{x} - 9$$

by <u>the method of undetermined coefficients (annihilator approach)</u>.

Q6. (**17 points**) Solve the boundary value problem

$$y'' + y = \cot x, \quad y\left(\frac{\pi}{4}\right) = 0, \quad y\left(\frac{\pi}{2}\right) = 0.$$

Q7. (**16 points**) Solve the differential equation

$$x^2 y'' - x\,y' - 3\,y = \ln x$$

on the interval $(0,\infty)$.

Q8. (**14 points**) The function $y_1 = e^x$ is a solution of the associated homogeneous equation of

$$xy'' - 2y' + (2 - x)y = x^3.$$

Use <u>the method of reduction of order</u> to find the general solution of the given differential equation.