Thursday, Nov. 23, 2017  Allowed Time: 2 Hours

Name: __________________________________________

ID Number: ___________  Serial Number: _____________

Section Number: ________  Instructor’s Name: __________

Instructions:

1. Write neatly and legibly. You may lose points for messy work.

2. Show all your work. No points for answers without justification.

3. Calculators and Mobiles are not allowed.

4. Make sure that you have 8 different problems (10 pages).

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<th>Problem No.</th>
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Q1. (7 points) Use the **existence and uniqueness theorem** to find the **largest interval** so that the given initial value problem has a unique solution

\[(x - 2)y'' + \ln(x + 2)y = x, \quad y(0) = 0, \quad y'(0) = 1.\]
Q2. Let $y_1 = \cos(\ln x)$ and $y_2 = \sin(\ln x)$ be both solutions of the differential equation

$$x^2 y'' + x y' + y = 0.$$ 

(a) (7 points) Use the Wronskian to verify that $y_1$ and $y_2$ form a fundamental set of solutions of the given differential equation on the interval $(0, \infty)$.

(b) (5 points) Use part (a) to verify that $y = c_1 \cos(\ln x) + c_2 \sin(\ln x) + 1$ is the general solution of the nonhomogeneous differential equation

$$x^2 y'' + x y' + y = 1.$$
Q3. (6 points) Given that $y_1 = -2 - 2e^x(x + 3)$ and $y_2 = \frac{3}{2} - e^{-x}(x + 1)$ are, respectively, particular solutions of the differential equations $y'' - 2y = 4 + 2xe^x$ and $y'' - 2y = -3 + 3xe^{-x}$, find a particular solution of the differential equation

$$y'' - 2y = \frac{1}{2} + x \cosh x.$$
Q4. (13 points) Given that $y_1 = xe^{5x}$ is a solution of

$$y^{(4)} - 12y^{(3)} + 47y'' - 70y' + 50y = 0,$$

find the general solution of the given differential equation.
Q5. (15 points) Solve the differential equation

\[ y'' - 2y' - 3y = 4e^x - 9 \]

by the method of undetermined coefficients (annihilator approach).
Q6. (17 points) Solve the boundary value problem

\[ y'' + y = \cot x, \quad y\left(\frac{\pi}{4}\right) = 0, \quad y\left(\frac{\pi}{2}\right) = 0. \]
Q7. (16 points) Solve the differential equation

\[ x^2 y'' - x y' - 3y = \ln x \]

on the interval \((0, \infty)\).
Q8. (14 points) The function \( y_1 = e^x \) is a solution of the associated homogeneous equation of
\[ xy'' - 2y' + (2 - x)y = x^3. \]

Use the method of reduction of order to find the general solution of the given differential equation.