Name: ___________________________  ID Number: ___________________________
Section Number: ________________  Serial Number: ____________________________
Class Time: ______________________  Instructor’s Name: ______________________

Instructions:

1. Calculators and Mobile Phones are not allowed.
2. Please write legibly.
3. Show all your work. No points for answers without justification.
4. Make sure that you have 7 pages of problems (Total of 7 Problems)

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<th>Question # Number</th>
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1. [8 points] (a) Determine whether the three vectors \( \mathbf{u} = (1, 2, 3) \), \( \mathbf{v} = (4, 5, 6) \) and \( \mathbf{w} = (7, 8, 9) \) are linearly independent or linearly dependent?

[4 points] (b) Do vectors in part(a) form a basis of \( \mathbb{R}^3 \)? Justify your answer.
2. [10 points] (a) Find basis for the solution space of the system

\[ \begin{align*}
    x + 4y + 7z &= 0, \\
    2x + 5y + 8z &= 0, \\
    3x + 6y + 9z &= 0.
\end{align*} \]

(Note: To answer this question, you can use your calculations in question 1 part(a).)

[b] [3 points](b) Give the dimension of the solution space of the system in part(a).
3. [6 points] (a) Consider a set $W$ of all vectors $(\frac{1}{2}, y, z)$ in $\mathbb{R}^3$. Determine whether this set forms a subspace?

[9 points] (b) Consider a set $W$ of all vectors in $\mathbb{R}^3$ such that $x + z = 2y$. Determine whether this set forms a subspace or not?
4. [5 points] (a) Verify that the solutions $y_1 = x$, $y_2 = x^2$ and $y_3 = x^3$ are linearly independent solutions of the differential equation $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0$ on the interval $(0, \infty)$.

(b) [11 points] Solve the IVP

$$x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0,$$

$$y(1) = 2, \ y'(1) = 3, \ y''(1) = 4.$$
5. [13 points] The general solution of a constant coefficient, linear homogeneous differential equation is $y = Ae^{2x} + B \cos x + C \sin x$. Find the corresponding differential equation.
6. **[15 points]** Find general solution of the constant coefficient homogeneous differential equation \(D^2(D - 3)(D^2 + 2D + 2)y = 0\).
7. [16 points] Use variation of parameters method to solve the differential equation 
\[ y'' - y = e^{2x} - e^{-2x}. \] The solutions of the associated homogeneous equation are 
\[ y_1 = e^x \] and 
\[ y_2 = e^{-x}. \]