1. (12 pts) Let $a, b$ and $c$ be real numbers. Show that $a(-b) = -(ab) = (-a)b$.

2. (12 pts) Prove that $\sqrt{n} + 1$ is irrational number for every natural number $n$.

3. (24 pts) If $S$ is the set given by

$$S = \left\{ \frac{1}{3} \pm \frac{n}{3n+1} \mid n \in \mathbb{N} \right\},$$

prove that $\inf S = 0$ and $\sup S = \frac{2}{3}$.

4. (12 pts) Show that the sequence $\left(\left(1 + \frac{1}{n}\right)^n\right)$ is increasing.

5. (12 pts) Let $(x_n)$ be a sequence of real numbers defined as

$$x_1 = \frac{1}{4} \quad \text{and} \quad x_{n+1} = \frac{1}{2} \left(1 + x_n^2\right).$$

Show that $\lim (x_n)$ exists and find it.

6. (12 pts) If $(x_n)$ is a convergent sequence of positive real numbers, using the definition of a convergent sequence prove that the sequence $(\ln x_n)$ is convergent.

7. (12 pts) Using $\epsilon - \delta$ definition of limit show that

$$\lim_{x \to -1} \frac{x + 5}{2x + 3} = 4.$$

8. (12 pts) Find $\lim (x_n)$ where

$$x_n = \ln \left(1 - \frac{1}{2^2}\right) + \ln \left(1 - \frac{1}{3^2}\right) + \cdots + \ln \left(1 - \frac{1}{n^2}\right), \quad n \geq 2.$$

**Hint:** show that

$$\prod_{i=2}^{n} \left(1 - \frac{1}{i^2}\right) = \frac{1}{2} \left(1 + \frac{1}{n}\right).$$

9. (12 pts) Find the limit

$$\lim_{x \to 1} \frac{(1 - \sqrt{x})(1 - \sqrt[3]{x}) \cdots (1 - \sqrt[n]{x})}{(1 - x)^{n-1}}, \quad n \in \mathbb{N}.$$

**Hint:** you may consider the substitution $t = 1 - x$ and show that

$$\lim_{t \to 0} \frac{1 - \sqrt[n]{1-t}}{t} = \frac{1}{n}, \quad \text{for all} \ n \in \mathbb{N}.$$