King Fahd University of Petroleum and Minerals
Department of Mathematics & Statistics
Math 465 Final Exam
The First Semester of 2017-2018 (171)
Time Allowed: 120mn

Name: ID number:

Textbooks are not authorized in this exam

Problem #	Marks	Maximum Marks
1		20
2		30
3		25
4		25
Total		100

Problem 1: Consider the system

$$\frac{dx}{dt} = -x \qquad (1)$$

$$\frac{dy}{dt} = -2y - \alpha y^2 \qquad (2)$$

Sketch the trajectories of the system that passe through thepoint $(x,y)=(1,1)$ for $\alpha=0$ and $\alpha=1$.

<u>Solution</u>

$$\frac{dy}{dx} = \frac{2y+\alpha y^2}{x}$$

$$\Leftrightarrow \int \frac{dy}{2y+\alpha y^2} = \int \frac{dx}{x}$$

$$\int \left(\frac{1}{2y} - \frac{\alpha}{2(2+\alpha y)}\right) dy = \ln|x| + c$$

$$\frac{1}{2}\left[\ln|y| - \ln|2+\alpha y|\right] = \ln|x| + C$$

$$\frac{y}{2+\alpha y} = Cx^2$$

a) $\alpha = 0$, $y = Cx^2$

$x=1, y=1 \Rightarrow C=1$

$$\boxed{y = x^2, \; x \in (-\infty, \infty)}$$

b) $\alpha = 1$, $y = \dfrac{2Cx^2}{1-Cx^2}$

$x=1, y=1 \Rightarrow 1 = \dfrac{2C}{1-C}$

$\Rightarrow C = 1/3$

$$\boxed{y = \frac{\frac{2}{3}x^2}{1-\frac{1}{3}x^2}, \; x \in (-\sqrt{3}, \sqrt{3})}$$