

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

DEPARTMENT OF MATHEMATICS & STATISTICS

AS 498 – Term 172

Date: March 04, 2018

Exam 1

Duration: 120 minutes

Name:

ID #: _ _ _ _ _

Section #: 01

Serial #: _ _

Instructions:

- Write neatly and eligibly. You may lose points for messy work.
- Show all your work. No points for answers without justification.
- **Electronic approved calculators** are allowed. Mobiles are NOT allowed.
- Make sure that you have **8** pages of problems (Total of **6** Questions).

Question #	Score
1	/ 10
2	/ 10
3	/ 10
4	/ 15
5	/ 15
6	/ 20
Total	/ 80

1. If the continuously compounded risk-free interest rate is 4%, calculate the 6-month profit for the following investors:

(a) **Investor A** short-sell 100 shares of stock X . The current price of stock X is 48 per share and the company has announced that it is going to pay a dividend of 0.5 per share after 1 month and 4 months. After 6 months, he covered the short position when the stock price was 50.

(b) **Investor B** purchases 100 units of stock index Y , which is currently priced at 45 and pays dividends continuously at a rate proportional to its price at a constant rate of 2%. The investor invested all dividends into the index and after 6 months she closed out all positions when the stock index price was 44.

(5 + 5 = 10 points)

2. For 4-month European options, you are given:
- (i) The underlying stock pays no dividends.
 - (ii) The continuously compounded risk-free interest rate is 10%.
 - (iii) The price of a 30-strike call is 1.5 higher than the price of a 32-strike call.

Calculate the amount by which the price of an otherwise equivalent 32-strike put exceeds the price of an otherwise equivalent 30-strike put. **(10 points)**

3. You consider a single-period binomial model for a 1-month at-the-money dollar denominated European put option on British pounds. You are given:

- (i) The current exchange rate is 1.5 US dollar per British pounds.
- (ii) In one month, the exchange rate either moves up by a proportional amount of 15% or moves down by a proportional amount of 25%.
- (iii) The US dollar continuously compounded risk-free interest rate is 7%.
- (iv) The British pound continuously compounded risk-free interest rate is 9%.

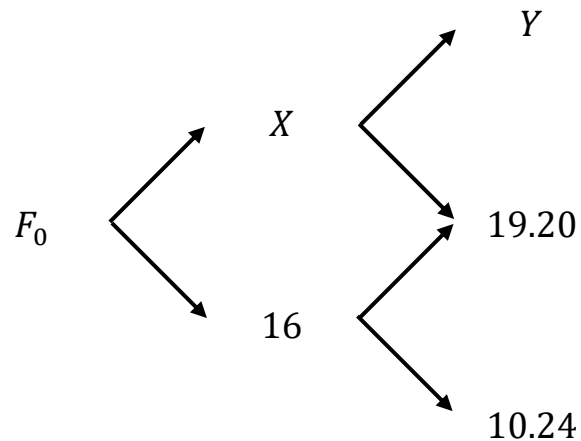
(a) Construct a replicating portfolio for the put option.

(b) Calculate the current price of the put option.

(7 + 3 = 10 points)

4. You are to price options on a futures contract. You are given:

- (i) The movements of the futures price can be modeled by the following (incomplete) binomial tree.



- (ii) The length of each period is 1 month.

- (iii) The continuously compounded risk-free interest rate is 5%.

- (a) Calculate the price of a 2-month at-the-money European call option on a futures contract.

- (b) Calculate the price of a 2-month at-the-money European put option on a futures contract.

(11 + 4 = 15 points)

5. For a stock, you are given:
- (i) The current stock price is 80.
 - (ii) A dividend of 0.5 per share will be paid after 2 months and 5 months.
 - (iii) The theoretical forward price on the stock that matures after 4 months is 81.51892.
- (a) Calculate the continuously compounded risk-free interest rate.

(b) Calculate the prepaid forward price of a prepaid forward contract that delivers one share of the stock after 6 months.

(c) Construct an arbitrage portfolio if the market prepaid forward price is 78.

(5 + 3 + 7 = 15 points)

6. You use a Cox-Ross-Rubinstein tree to price derivatives on a nondividend-paying stock. You are given:

- (i) The length of each period is 3 months.
- (ii) The stock currently sells for 100.
- (iii) The stock's volatility is 30%.
- (iv) The continuously compounded risk-free interest rate is 8%.

Calculate the following:

(a) The risk-neutral probability of an up-move.

(b) The price of a 6-month 90-strike European put option on the stock.

(c) The price of a 6-month 90-strike American put option on the stock.

(d) The price of a 6-month 90-strike straddle on the stock that has the payoff $|90 - S(0.5)|$ and will be exercised only at the end of 6-month.

(4 + 5 + 6 + 5 = 20 points)