KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DEPARTMENT OF MATHEMATICS & STATISTICS

AS498 - Section 01 (Term 172)

Date: May 03, 2018 **Test 5** Duration: 50 minutes

Family Name: _____ **ID #:** 2 0 1 _ _ _ _ _ _ 0 **Serial #:** _ _

1. Let $S(t)$ denote the price at time t of a stock. Consider an 8-month European gap option. If the stock price after 8 months is less than 28, the payoff is $28.5 - S(8/12)$; otherwise the payoff is zero. You are given:

   (i) $S(0) = 30$.
   (ii) The stock will pay a dividend of 2 dollars after 4 months. This is the only dividend to be paid in the coming 8 months.
   (iii) The prepaid forward price of the stock follows the Black-Scholes model with a volatility of 33%.
   (iv) The continuously compounded risk-free interest rate is 10%

   Calculate the price of the gap option. **(8 points)**

2. For two stocks S and Q, you are given:

   (i) The current stock price of S is 75.
   (ii) The current stock price of Q is 100.
   (iii) Both stocks pay dividends continuously at a rate proportional to its price. The dividend yields for S and Q are 3% and 5%, respectively.
   (iv) The current price of an option to exchange 3 units of Q for 4 units of S at time 1 is 86.28.

   Compute the price of an option to exchange 16 units of S for 12 units of Q at time 1.

   (6 points)

3. You are given:

   (i) Stock S is a nondividend-paying stock that is currently priced at 30.
   (ii) Stock Q is currently priced at 60. It pays dividends at a rate that is proportional to its price. The continuous dividend yield is 4%.
   (iii) The volatility of S and Q are 0.3 and 0.5.
   (iv) The correlation between the continuously compounded returns on S and Q is -0.4.
   (v) The continuously compounded risk-free interest rate is 10%.

   Calculate the current price of a derivative that gives the option holder the right to exchange 1 share of Q for 2 shares of S after 3 months. **(10 points)**

4. You are given the following information on prices of European put options with the same underlying stock and maturity:

Strike	60	63
Put Price	17	20.5

Show what transactions that can construct an arbitrage opportunity. **(6 points)**

5. You are given the following information on prices of European call options with the same underlying stock and maturity:

Strike	38	43	48
Call Price	14.45	10.75	7

Show what transactions that can construct an arbitrage opportunity. **(8 points)**

6. For a stock, you are given:

   (i) The current stock price is 45.
   (ii) The stock is going to pay a dividend of 1 after 3 months. This is the only dividend to be paid in the coming 6 months.
   (iii) A 6-month 42-strike American call option on the stock has a premium of 0.36.
   (iv) The continuously compounded risk-free interest rate is 5%

   Consider a 6-month 42-strike American put option on the stock. Would it be possible that exercising the American put option now is optimal? **(5 points)**

7. For a stock, you are given:

   (i) The current stock price is 100.
   (ii) The stock is going to pay a dividend of 1.5 after 6 months. This is the only dividend to be paid in the coming 9 months.
   (iii) The continuously compounded risk-free interest rate is 5%

   Consider a 9-month K-strike American put option on the stock. Given the only information above, find the range of values of K so that it is certain that it is NOT optimal to exercise the K-strike American put. **(7 points)**