

Serial No.: \_\_\_\_\_ Student Name: \_\_\_\_\_ Student Number: \_\_\_\_\_

Instructor: M. Z. Abu-Sbeih

Math 101- Q5

Date: 26-4-2018

**SHOW ALL YOUR WORK. NO CREDITS FOR ANSWERES WITHOUT JUSTIFICATIONS**

**Problem 1** (6 points) Show that the function  $f(x) = x\sqrt{x-1}$  satisfies the hypothesis of Rolle's Theorem on the interval  $[0,1]$ . Find all values of  $c$  that satisfy the conclusion of the theorem in that interval.

**Problem 2** (12 points) Evaluate the limit if it exists. Use  $\infty$  and/or  $-\infty$  when appropriate.

a)  $\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x}$

b)  $\lim_{x \rightarrow 0^+} (1 + \sin x)^{\frac{1}{x}}$

**Question 3:** (24 points) Consider the function  $y = f(x) = \frac{x^2}{x-1} = (x+1) + \frac{1}{x-1}$  with

$$f'(x) = \frac{x^2 - 2x}{(x-1)^2} \text{ and } f''(x) = \frac{2}{(x-1)^3}$$

- a. (2 Points) Find the intercepts.
- b. (3 Points) Find the asymptotes if any exist. **(if no asymptote, say no asymptote)**  
**Horizontal:**  
  
**Vertical:**  
  
**Slant:**
- c. (2 Points) Find the critical numbers.
- d. (4 Points) Find intervals where the function is increasing and those where it is decreasing.
- e. (4 Points) Find the local maximum and local minimum of the function.

f. (3 Points) Discuss the concavity of the function and find the inflection points if any exist.

g. (6 Points) Sketch the graph of the function. Clearly indicate the **critical numbers**, **extrema** and **inflection points** on the graph.

