Q1. Use limits to determine whether or not \( x = 0 \) is a vertical asymptote of \( f(x) = \frac{x^2 + x}{x^3 - 6x^2} \).

Solution:

\[
\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^2 + x}{x^3 - 6x^2}
\]

\[
= \lim_{x \to 0} \frac{x(x + 1)}{x(x^2 - 6x)}
\]

\[
= \lim_{x \to 0} \frac{x + 1}{x^2 - 6x}
\]

\[
= \lim_{x \to 0} \frac{1}{x - 6}
\]

\[
= \frac{0}{-6} = 0
\]

The limit exists and is equal to 0 from both the right and left sides. Therefore, \( x = 0 \) is a vertical asymptote.

Q2. Evaluate \( \lim_{x \to -\sqrt{3}} \left[ \frac{1}{4-x^2} \right] \) if it exists and explain if it does not. (where \( \lfloor x \rfloor \) is the greatest integer \( \leq x \)).
Q1. **Use limits** to determine whether or not $x = 1$ is a vertical asymptote of $f(x) = \frac{x^2 - 2x + 1}{4x^2 + 4x - 8}$.

\[
\lim_{{x \to 1}} f(x) = \lim_{{x \to 1}} \frac{x^2 - 2x + 1}{4x^2 + 4x - 8} = \lim_{{x \to 1}} \frac{(x - 1)(x - 1)}{4(x - 1)(x + 2)} = \lim_{{x \to 1}} \frac{x - 1}{4(x + 2)}
\]

\[
= \frac{1 - 1}{4(1 + 2)} = 0.
\]

So, $x = 1$ **is NOT** a vertical asymptote.

Q2. Evaluate $\lim_{{x \to 0}} \frac{x}{2 - \sqrt{4 + x}}$ if it exists and **explain if it does not**.