Math201.01, Quiz #2, Term 172

Name:

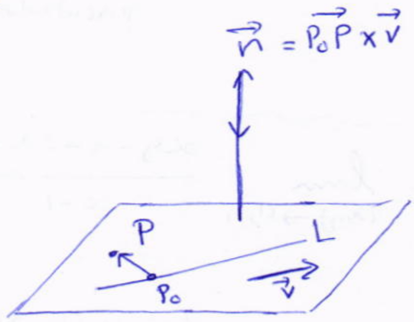
Solutions

ID#: **Serial #:**

1. **[3 points]** Find an equation of the plane that contains the point $(1,0,-2)$ and the line $x = 3t, y = 1+t, z = 2-t$.
2. **[3 points]** Identify (name, vertex, axis) and sketch the surface $x^2 - 2x + 2y^2 + 8y - z = -12$.
3. **[2 points]** Find and sketch the domain of $f(x,y) = \frac{\ln(x-1)}{y-x^2}$.
4. **[2 points]** Find the limit if it exists: $\lim_{(x,y)\to(1,1)} \frac{xy-y-2x+2}{x-1}$.

Good luck,

Ibrahim Al-Rasasi



$\boxed{1}$. $P(1,0,-2)$ a point in the plane.

. $t=0 \Rightarrow P_0(0,1,2)$ $\underline{\underline{0.5}}$

. a vector parallel to the line: $\vec{v} = \langle 3,1,-1\rangle$. $\underline{\underline{0.5}}$

. The normal vector to the plane is

$\underline{\underline{0.5}}$ $\vec{n} = \vec{P_0P} \times \vec{v}$, $\vec{P_0P} = \langle 1,-1,-4\rangle$ $\underline{\underline{0.5}}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -4 \\ 3 & 1 & -1 \end{vmatrix}$$

$\underline{\underline{0.5}}$ $= 5\vec{i} - 11\vec{j} + 4\vec{k}$

. an equation of the plane is

$$5(x-1) - 11(y-0) + 4(z+2) = 0$$

$$5x - 11y + 4z - 5 + 8 = 0$$

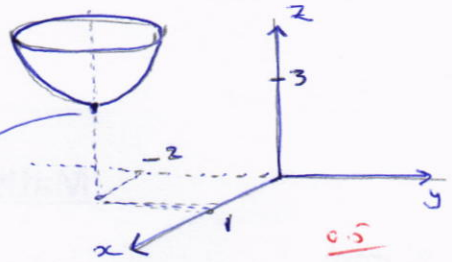
$$5x - 11y + 4z = -3$$ $\underline{\underline{0.5}}$

[2] $x^2 - 2x + 2y^2 + 8y - z = -12$

$x^2 - 2x + 1 + 2(y^2 + 4y + 4) = z - 12 + 1 + 8$

$(x-1)^2 + 2(y+2)^2 = z - 3$

or $z = (x-1)^2 + 2(y+2)^2 + 3$ 1

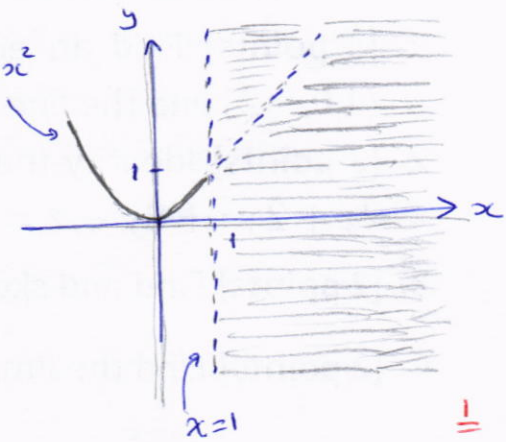


0.5 . an elliptic paraboloid

0.5 . vertex : $(1, -2, 3)$

0.5 . axis : the line through the vertex $(1, -2, 3)$ and parallel to the z-axis.

[3] $f(x,y) = \dfrac{\ln(x-1)}{y - x^2}$



Domain $= \{(x,y) : x - 1 > 0 \text{ and } y - x^2 \neq 0\}$

$= \{(x,y) : x > 1 \text{ and } y \neq x^2\}$ 1

= all points (x,y) to the right of the line $x = 1$ and not on the parabola $y = x^2$.

[4] $\lim\limits_{(x,y)\to(1,1)} \dfrac{xy - y - 2x + 2}{x - 1}$, $\frac{0}{0}$, undefined

Factor : $xy - y - 2x + 2$
$= y(x-1) - 2(x-1)$
$= (x-1)(y-2)$ 1

$= \lim\limits_{(x,y)\to(1,1)} \dfrac{(x-1)(y-2)}{x-1}$

$= \lim\limits_{(x,y)\to(1,1)} y - 2$ 0.5

$= 1 - 2 = -1.$ 0.5

Math201.02, Quiz #2, Term 172

Name:

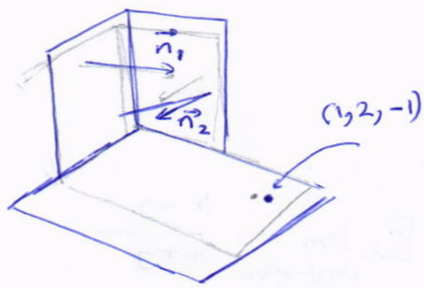
Solutions

ID#: **Serial #:**

1. **[3 points]** Find an equation of the plane that passes through the point $(1,2,-1)$ and is perpendicular to the planes $2x+y-2z=2$ and $x+3z=4$.
2. **[3 points]** Identify (name, vertex, axis) and sketch the surface $2x^2+4x+y^2-2y-z^2=-3$.
3. **[2 points]** Find and sketch the domain of $f(x,y)=\sqrt{xy-1}$.
4. **[2 points]** Find the limit if it exists: $\lim\limits_{(x,y)\to(0,0)} \frac{x-y}{x+y}$.

Good luck,

Ibrahim Al-Rasasi



$\boxed{1}$ $P_1: 2x+y-2z=2 \implies \vec{n}_1=\langle 2,1,-2\rangle$ 0.5

$P_2: x+3z=4 \implies \vec{n}_2=\langle 1,0,3\rangle$ 0.5

- a normal vector to the required plane is

$$\vec{n}=\vec{n}_1\times\vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

1

$$= 3\vec{i}-8\vec{j}-\vec{k} \ \text{or}\ \langle 3,-8,-1\rangle$$ 0.5

- an equation of the plane is

$$3(x-1)-8(y-2)-(z+1)=0$$

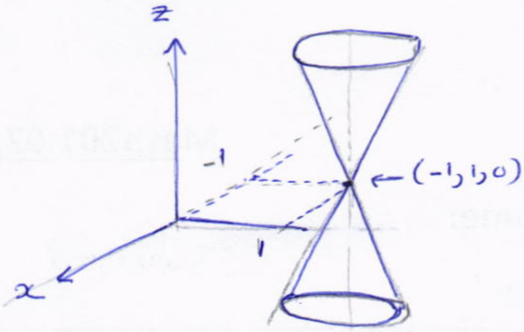
$$\implies 3x-3-8y+16-z-1=0$$

$$\implies 3x-8y-z=-12$$ 0.5

[2] $2x^2+4x+y^2-2y-z^2=-3$

$2(x^2+2x+1)+(y^2-2y+1)=z^2-3+2+1$

$2(x+1)^2+(y-1)^2=z^2$ 1



- 0.5 an elliptic cone
- 0.5 vertex : $(-1,1,0)$
- 0.5 axis : the line through the vertex $(-1,1,0)$ and is parallel to the z-axis

0.5

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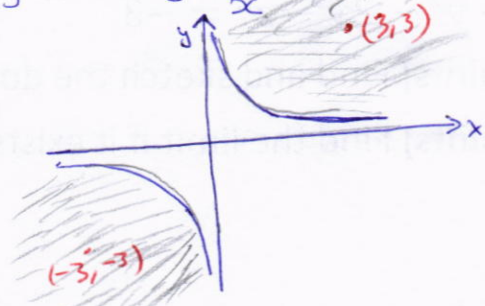
[3] $F(x,y)=\sqrt{xy-1}$

Domain $=\{(x,y): xy-1\geq 0\}$

$=\{(x,y): xy\geq 1\}$

1

$xy=1 \Rightarrow y=\frac{1}{x}$



1

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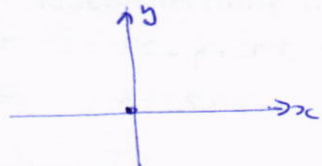
[4] $\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y}$



- along the x-axis : $y=0$

$$\lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{x-y}{x+y} = \lim_{(x,y)\to(0,0)} \left.\frac{x-y}{x+y}\right|_{y=0} = \lim_{(x,y)\to(0,0)} \frac{x-0}{x+0} = \lim_{(x,y)\to(0,0)} 1 = 1$$

- along the y-axis : $x=0$

$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}} \frac{x-y}{x+y} = \lim_{(x,y)\to(0,0)} \left.\frac{x-y}{x+y}\right|_{x=0} = \lim_{(x,y)\to(0,0)} \frac{0-y}{0+y} = \lim_{(x,y)\to(0,0)} -1 = -1$$

Since the limits along the two paths are <u>not</u> equal, then

$$\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y} \text{ DNE.}$$