Name:

KFUPM ID:

Exercise 1

Consider the following differential equation

\[ y' - y^2 + 3y - 2 = 0. \]  \hfill (1)

1. Show that

\[ y(x) = \frac{2 - c e^x}{1 - c e^x}, \]

is a one-parameter family of solutions of the differential equation \( (1) \).

2. Find a singular solution of the DE \( (1) \).
Exercise 2

Consider the following initial value problem

\[
\begin{align*}
    y' &= \sqrt{y^2 - 4} + \sqrt{9 - x^2}, \\
    y(x_0) &= y_0.
\end{align*}
\]  

(2)

1. Find and sketch the region of all \((x_0, y_0) \in \mathbb{R}^2\) such that the initial value problem (2) has a unique solution following the Theorem of existence uniqueness of solutions.

2. Find the largest interval on which the solution of the IVP (2) with \(x_0 = 1\) and \(y_0 = 3\) may be defined.
Exercise 3

Solve the following initial value problem

\[
\begin{cases}
y' - xe^{x+y} = 0, \\
y(0) = 0,
\end{cases}
\]

and find the largest interval on which the solution may be defined.
Exercise 4

1. Evaluate the derivative of the function \( 1 + (\ln x)^2 \).

2. Solve the following differential equation

\[
x (1 + (\ln x)^2) \frac{dy}{dx} + 2 \ln(x) \cdot y = 1,
\]

on \((0, +\infty)\).