1. Show that $y_1 = \sin x$ and $y_2 = \cos x$ form a fundamental set of solutions for the differential equation $y'' + y = 0$ on $(-\infty, \infty)$.

2. If $y_1 = e^x$ is a solution of the associated homogeneous equation of $y'' - 3y' + 2y = 5e^{3x}$, then find its second solution $y_2$ and the general solution of this equation.
1. Check whether or not the following BVP has a solution.

\[ y'' + 4y = 0, \quad y(0) = 0, \quad y(\pi) = 1. \]

2. If one of the solutions of \( xy'' - y' + 4x^2y = 0 \) is \( y_1(x) = \sin(x^2) \), then find its other solution \( y_2(x) \). Calculate value of its general solution \( y(x) \) at \( x = 0 \).
2. $y_1 = x^2$ is a solution of $x^2y'' - 3xy' + 4y = 0$. Find the general solution of this equation on $(0, \infty)$.

3. Find a homogeneous linear differential equation of smallest order for which the fundamental set of solutions is \( \{1, x, e^{-3x}, e^{2x} \cos 5x, e^{2x} \sin 5x\} \).