MATH 202.14 (Term 172)

Quiz 2 (Sects. 2.4 & 2.5) Duration: 20min

Name: ID number:

1.)(5pts)Solve the DE: $(-2ye^x + x^3 + \ln y)\frac{dy}{dx} = y^2e^x + x\sin^2 x - 3x^2y$.

2.)(5pts)Solve the DE: $(1+e^{2x})\frac{dy}{dx} + \frac{3}{2}(1+e^{2x})y = y^{1/3}$.

1.) $M(x,y) = y^2e^x + x\sin^2 x - 3x^2y$

$N(x,y) = 2ye^x - x^3 - \ln y$

$M_y = 2ye^x - 3x^2$

$N_x = 2ye^x - 3x^2$

$M_y = N_x$ $\Rightarrow$ Exact DE

$\frac{\partial f}{\partial x} = M = y^2e^x + x\sin^2 x - 3x^2y$

$\Rightarrow f(x,y) = y^2e^x + \underbrace{\int x\sin^2 x\,dx} - x^3y + g(y)$

$u = x \rightarrow u' = 1$ (integration by parts)

$v' = \sin^2 x = \frac{1-\cos 2x}{2} \rightarrow v = \frac{x}{2} - \frac{\sin 2x}{4}$

$\Rightarrow f(x,y) = y^2e^x + \left(\frac{x^2}{4} - x\frac{\sin 2x}{4} - \frac{\cos 2x}{8}\right) - x^3y + g(y)$

Now, $2ye^x - x^3 + g'(y) = 2ye^x - x^3 - \ln y$

$g'(y) = -\ln y$

$g(y) = \underbrace{-\int \ln y\,dy}_{\text{integration by parts}}$

$u = \ln y \rightarrow u' = \frac{1}{y}$

$v' = 1 \rightarrow v = y$

$\Rightarrow g(y) = y - y\ln y$

$$\boxed{y^2e^x + \left(\frac{x^2}{4} - x\frac{\sin 2x}{4} - \frac{\cos 2x}{8}\right) - x^3y + y - y\ln y = c}$$

2.) This is Bernoulli's DE

$u = y^{1-\frac{1}{3}} = y^{\frac{2}{3}}$, $y = u^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}u^{\frac{1}{2}}\frac{du}{dx}$

$(1+e^{2x})\frac{3}{2}u^{1/2}\frac{du}{dx} + \frac{3}{2}(1+e^{2x})u^{3/2} = u^{\frac{1}{2}}$

$\frac{du}{dx} + u = \frac{2}{3}\frac{1}{1+e^{2x}}$

an integrating factor is e^x

$\frac{d}{dx}(ue^x) = \frac{2}{3}\frac{e^x}{1+e^{2x}}$

$ue^x = \frac{2}{3}\underbrace{\int\frac{e^x}{1+e^{2x}}dx}_{\text{Substitution}}$

$v = e^x$, $dv = e^x dx$

$\int\frac{dv}{1+v^2} = \tan^{-1} v$

$\Rightarrow ue^x = \frac{2}{3}\tan^{-1}(e^x) + c$

$$\boxed{y^{\frac{2}{3}} = e^{-x}\left(\frac{2}{3}\tan^{-1}e^x + c\right)}$$