(1) (a) Prove or disprove: If \( \{f_k\}_{k=1}^\infty \) is uniformly integrable over a set \( E \) and \( \{f_n\} \to f \) pointwise a.e. on \( E \), then \( f \) is integrable over \( E \).

(b) Let \( \mathcal{F} \) be a family of functions, each of which is integrable over \( E \). Show that \( \mathcal{F} \) is uniformly integrable over \( E \) if and only if for each \( \varepsilon > 0 \), there is a \( \delta > 0 \) such that for each \( f \in \mathcal{F} \),

\[
\text{if } A \subseteq E \text{ is measurable and } m(A) < \delta, \text{then } \left| \int_A f \right| < \varepsilon.
\]
(2) (a) Let \( \{f_n\} \to f \) in measure on \( E \) and \( g \) be a measurable function on \( E \) that is finite a.e. on \( E \). Show that \( \{f_n\} \to g \) in measure on \( E \) if and only if \( f = g \) a.e. on \( E \).

(b) Let \( f \) and each \( f_n \) be integrable and \( \int |f_n - f| \, dm \to 0 \). Show that \( f_n \to f \) in measure.
A real-valued function $f$ defined on an interval $[a, b]$ satisfies a Lipschitz-condition with constant $M$ if $|f(y) - f(x)| \leq M|y - x|$ for all $x, y \in [a, b]$. Prove that $f$ satisfies a Lipschitz condition with constant $M$ if and only if

(i) $f$ is absolutely continuous on $[a, b]$, and

(ii) $|f'(x)| \leq M \text{ m} - a.e.$
(4) (a) Let $f$ be integrable over $[0, 1]$. Show that

$$exp\left[\int_0^1 f(x) \, dx\right] \leq \int_0^1 exp(f(x)) \, dx$$

(b) Compute $TV_{[0,50]}(e^x)$, the total variation of $e^x$ on the interval $[0, 50]$. 

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