1. Compute
\[ \int_0^\infty \frac{x^{1/3}}{1 + x^2} \, dx. \]

2. Let \( f : G \to \mathbb{C} \) be a meromorphic function with zeros \( a_1, \ldots, a_m \) and poles \( p_1, \ldots, p_l \) in \( G \) (repeated according to multiplicity). Let \( \gamma \) be a closed curve in \( G \) which does not pass through any \( a_j \) or any \( p_k \). Suppose \( \gamma \approx 0 \) in \( G \). Then prove that
\[ \frac{1}{2\pi i} \int_{\gamma} \frac{z^n f'(z)}{f(z)} \, dz = \sum_{j=1}^m n(\gamma; a_j)a_j^n - \sum_{k=1}^l n(\gamma; p_k)p_k^n \]
for any positive integer \( n \).

3. Prove or disprove the following statements.
   (a) Let \( f : G \to \mathbb{C} \) be an analytic function such that \( f'(a) = 0 \) for some \( a \in G \). Then \( f \) cannot be 1-1 on any neighborhood of \( a \).
   (b) If \( f : G \to \mathbb{C} \) is not 1-1, then there exists a point \( a \in G \) such that \( f'(a) = 0 \).

4. Find the number of zeros of \( f(z) = z^8 - 5z^2 e^{z+1} + 4z \) in \( D \).

5. Let \( U \) be a bounded open region in \( \mathbb{C} \) and let \( \{ f_n : G \to U \} \) be a sequence of analytic function into \( U \). Suppose that there exists a point \( a \in G \) and \( p \in \partial U \) such that
\[ \lim_{n \to \infty} f_n(a) = p. \]
Prove that for any compact subset \( K \) of \( G \) and for any \( \varepsilon > 0 \), there exists \( N > 0 \) such that
\[ f_n(K) \subset B(p; \varepsilon) \]
if \( n \geq N \).

6. Let \( f : D \to D \) be analytic. Suppose that \( f(0) = 0 \) and \( f'(0) = a \).
   (a) Let \( g(z) := f(z)/z \). Then show that \( g \) is an analytic function from \( D \) into \( D \) and \( g(0) = a \).
   (b) For the \( g \) as above, show that \( g'(0) = f''(0)/2 \).
   (c) Using (a) and (b), prove \(|f''(0)| \leq 2(1 - |a|^2)\).
   (d) What is \( f \), if we further assume that \( f''(0) = 2(1 - |a|^2) \)?
7. Let $G = \mathbb{C} \setminus \{0\}$ and let $f \in \text{Aut}(G)$.
   
   (a) Show that $z = 0$ is either a removable singularity or a pole of $f$.
   
   (b) Show that either $f(z) = az$ or $f(z) = a/z$ for some nonzero constant $a$.

8. Let $u : H = \{z \in \mathbb{C} : \text{Im } z > 0\} \to \mathbb{R}$ be a harmonic function which is continuous up to $\partial H = \mathbb{R}$. Suppose $u$ is bounded and $u \equiv 0$ on $\partial H$. Prove $u \equiv 0$ on $H$. 