Problem 1 (40 pts)

Given the following linear program (P):

\[
\begin{align*}
\text{max} & \quad 2x_1 + 3x_2 \\
\text{s.t.} & \quad x_1 + 4x_2 \leq 5, \\
& \quad 5x_1 + 2x_2 \leq 7, \\
& \quad 2x_1 + x_2 \geq 2, \\
& \quad x_1, x_2 \geq 0.
\end{align*}
\]

(a) Solve the linear program (P) graphically. (10 points)

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1Dr. Slim Belhaiza (c)
2This is NOT an open book exam. The exam lasts 90 minutes.
(b) What is the impact on the optimal solution of an increase of the right hand side of the second condition by one unit? (10 points)

(c) For your interpretation in (b) to remain valid, how much is the maximum increase in the right hand side of the second condition? (5 points)

(d) Solve the linear program \((P)\) using the Simplex algorithm. (15 points)
Problem 2 (30 pts)

Consider the following linear program:

\[
\begin{align*}
\text{max} & \quad 2x_1 + 3x_2 + 2x_3 \\
\text{s.t.} & \quad 3x_1 + 3x_2 + x_3 \leq 7, \\
& \quad 2x_1 + x_2 + 3x_3 \leq 6, \\
& \quad x_1 + 2x_2 + 3x_3 \leq 6, \\
& \quad x_1, x_2, x_3 \geq 0.
\end{align*}
\]

Solve the linear program using the Tables’ representation of the Simplex algorithm. (30 points)

N.B.: You would be graded on 20 if you use any other representation.
**Problem 3 (30 Points)**

Consider the following linear program:

\[
\begin{align*}
\max_{x,y,z} & \quad x + 3y + 2z \\
\text{s.t.} & \quad 2x + 2y + z \leq 5, \\
& \quad x + 3y + z \leq 5, \\
& \quad x, y, z \geq 0.
\end{align*}
\]

Solve the linear program using the Matrices’ representation of the Simplex algorithm. (30 points)

N.B.: You would be graded on 20 if you use any other representation.