1. If $y = \cos 2x + x^{50}$ then $y^{(50)}\left(\frac{\pi}{2}\right) =$

(a) $\boxed{2^{50} + 50!}$

(b) $2^{49} + 50!$

(c) $-2^{49} + 50!$

(d) $-2^{50} + 50!$

(e) 2^{50}

let $f(x) = \cos 2x$

$f'(x) = -2\sin 2x$

$f''(x) = -4\cos 2x$

$f'''(x) = +8\sin 2x$

$f^{(4)}(x) = 16\cos 2x$

$\vdots$

$f^{(50)}(x) = -2^{50}\cos 2x$

$f^{(50)}\left(\frac{\pi}{2}\right) = -2^{50}\cos\pi$

$= 2^{50}$

let $g(x) = x^{50}$

$g'(x) = 50x^{49}$

$\vdots$

$g^{(50)}(x) = 50!$

$\Rightarrow g^{(50)}\left(\frac{\pi}{2}\right) = 50!$

So $y^{(50)}\left(\frac{\pi}{2}\right) = 2^{50} + 50!$

2. $\displaystyle\lim_{x\to 2^-} \frac{2+|x|}{2-[\![x]\!]} =$

(a) $\boxed{4}$

(b) 1

(c) does not exist

(d) ∞

(e) 0

$|x|$ $|-x|$ x, 0, 2

Note $\displaystyle\lim_{x\to 2^-} |x| = \lim_{x\to 2^-} x = 2$

$\displaystyle\lim_{x\to 2^-} [\![x]\!] = [\![2^-]\!] = 1$

$\displaystyle\lim_{x\to 2^-} \frac{2+|x|}{2-[\![x]\!]} = \frac{2+2}{2-1} = 4$

3. If the line $y = -2$ is a horizontal asymptote to the function

$$f(x) = \frac{ax^3 - 12}{(a+1)\,x^3 + 3x^2 - 9},$$

then $a =$

note: $\lim_{x \to \infty} \frac{ax^3 - 12}{(a+1)x^3 + 3x^2 - 9} = \frac{a}{a+1}$

(a) does not exist

$y = \frac{a}{a+1}$ is H.A

(b) 0

(c) -2

$\Rightarrow \frac{a}{a+1} = -2 \Rightarrow -2a - 2 = a$

(d) $\boxed{-\frac{2}{3}}$

$-3a = 2$

$a = \frac{-2}{3}$

(e) $\frac{1}{2}$

4. If the function

$$f(x) = \begin{cases} 7 + 3x \sin \frac{2}{x} & \text{if} \quad x \neq 0 \\ b - 1 & \text{if} \quad x = 0 \end{cases}$$

is continuous everywhere, then $b =$

f is cont at 0 $\Rightarrow \lim_{x \to 0} f(x) = f(0)$

(a) 6

$\lim_{x \to 0} 7 + 3x \sin \frac{2}{x} = b - 1$

(b) 13

(c) 7

$7 + 0 = b - 1 \Rightarrow \boxed{b = 8}$

(d) 5

(e) $\boxed{8}$

Note $\lim_{x \to 0} 3x \sin \frac{2}{x} = 0$ by the squeeze theorem

5. Let g be a twice differentiable function with $g'(4) = 8$ and $g''(4) = -3$. If $f(x) = x\,g(x^2)$ then $f''(2) =$

$\boxed{\text{(a) } 0}$

(b) 1

(c) -2

(d) -1

(e) 2

$f'(x) = g(x^2) + x\,g'(x^2).2x$

$= g(x^2) + 2x^2 g'(x^2)$

$f''(x) = g'(x^2).2x + 4x\,g'(x^2) + 2x^2 g''(x^2).2x$

$f''(2) = 4\,g'(4) + 8\,g'(4) + 32\,g''(4)$

$= \overset{12}{\cancel{\ }}\,g'(4) + 32\,g''(4)$

$= 12(8) + 32(-3)$

$= 96 - 96 = 0$

6. If the normal line to the parabola $f(x) = x^2 - 1$ at the point $(-1, 0)$ intersects the parabola a second time at the point (a, b), then $2b - a =$

$\boxed{\text{(a) } 1}$

(b) 2

(c) 3

(d) 0

(e) 4

$y = x^2 - 1 \Rightarrow y' = 2x$

at $(-1,0)$: $m_t = -2 \Rightarrow m_N = \frac{1}{2}$

the eqn of the normal line : $y - 0 = \frac{1}{2}(x+1)$

$\begin{cases} y = x^2 - 1 \\ y = \frac{1}{2}x + \frac{1}{2} \end{cases} \Rightarrow x^2 - 1 = \frac{1}{2}x + \frac{1}{2}$

$\Rightarrow x^2 - \frac{1}{2}x - \frac{3}{2} = 0$

$2x^2 - x - 3 = 0$

$(2x-3)(x+1) = 0$

$x = \frac{3}{2}$ | $x = -1$ given

$y = \left(\frac{3}{2}\right)^2 - 1$

$= \frac{9}{4} - 1 = \frac{5}{4}$

$\Rightarrow a = \frac{3}{2}$, $b = \frac{5}{4}$

$2b - a = \frac{5}{2} - \frac{3}{2} = 1$