1. [10pts] For each of the following DEs determine whether it is separable, 1st-order linear, or exact (justify your answers but do not solve the DEs):

(i) \((2y - \cos x)\, dx + (1 + x)\, dy = 0\).

(ii) \((3y + \sin 4x)\, dx + 3(e^y + x)\, dy = 0\).

(iii) \((4 + 2x + 2y + xy)\, dx + 4dy = 0\).

**Solution.** (i) DE can be rewritten as 
\[
\frac{dy}{dx} + \frac{2}{1+x}y = \frac{\cos x}{1+x}.
\]
So DE is 1st-order linear.

(ii) Let \(M = 3y + \sin 4x\) and \(N = 3(e^y + x)\). Then 
\[
\frac{\partial M}{\partial y} = 3 = \frac{\partial N}{\partial x}.
\]
So DE is exact.

(iii) DE can be rewritten as 
\[
\frac{dy}{dx} = -\frac{1}{4}(4 + 2x + 2y + xy),
\]
i.e. 
\[
\frac{dy}{dx} = -\frac{1}{4}(x + 2)(y + 2).
\]
Hence DE is separable.

[DE can also be written as 
\[
\frac{dy}{dx} + \frac{x + 2}{4}y = -\frac{1}{2}(x + 2),
\]
so this DE is 1st-order linear as well.]

2. [10pts] Solve the DE: \(xy' + 3y = 2x^5\).

**Solution.** In standard form DE is \(y' + \frac{3}{x}y = 2x^4\).

Integrating factor is \(\rho = e^{\int \frac{3}{x} \, dx} = x^3\).

Hence DE has family of solutions 
\[
y = \frac{1}{\rho} \left( \int \rho x^2 \, dx + C \right) = x^{-3} \left( \frac{x^5}{6} + C \right),
\]
i.e. 
\[
y = \frac{x^3}{6} + Cx^{-3}.
\]