Instructions.

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.

2. If you need to leave the room, please do so quietly so not to disturb others taking the test. No two person can leave the room at the same time. No extra time will be provided for the time missed outside the classroom.

3. Only materials provided by the instructor can be present on the table during the exam.

4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.

6. Only answers supported by work will be considered. Unsupported guesses will not be graded.

7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.

8. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators or financial calculators only. Write important steps to arrive at the solution of the following problems.

The test is 80 minutes, GOOD LUCK, and you may begin now!

<table>
<thead>
<tr>
<th>Question</th>
<th>Total Marks</th>
<th>Marks Obtained</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3+4+3=10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td></td>
<td></td>
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<tr>
<td>3</td>
<td>6</td>
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<tr>
<td>4</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2+4+4=10</td>
<td></td>
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</tr>
<tr>
<td>6</td>
<td>3+3+2=8</td>
<td></td>
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<tr>
<td>Total</td>
<td>45</td>
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</tbody>
</table>
Extra blank page
1. (3+4+3=10 points) $T_x$ and $T_y$ are independent future lifetime random variables, each with an exponential distribution with mean 20. Find:
   a) $E(T_{xy})$
   b) $\text{Var}(T_{xy})$.
   c) $\text{Cov}(T_{xy}, T_{xy})$.

2. (5 points) $K_{xy}$ is the curtate duration at failure of a joint-life status $(xy)$. You are given:

   1) $a_{xy} = 10$
   2) $2\bar{a}_{xy} = 7$
   3) $\text{Var}(\bar{a}_{K_{xy} + 1}) = 27$.

Find the discount rate $d$. 

3. (6 points) For a last-survivor whole life insurance of 1000 on \((x)\) and \((y)\):

1) Death benefit is payable at the moment of the second death.
2) The independent random variables \(T^*_x\), \(T^*_y\), and \(Z\) are the components of a common shock model.
3) \(T^*_x\) has an exponential distribution with a failure rate of 0.03, \(t \geq 0\).
4) \(T^*_y\) has an exponential distribution with a failure rate of 0.05, \(t \geq 0\).
5) The common shock random variable \(Z\) has an exponential distribution with a failure rate of 0.02, \(t \geq 0\).
6) \(\delta = 0.06\)

Calculate the **actuarial present value** of this insurance.

4. (6 points) Consider a risk free investment that pays $1000 one year from now, $3000 two years from now, and $5000 three years from now. Using the following interest rates:

<table>
<thead>
<tr>
<th>Scenario (j)</th>
<th>(j^1)</th>
<th>(j^2)</th>
<th>(j^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.01</td>
<td>0.05</td>
<td>0.09</td>
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<tr>
<td>3</td>
<td>0.09</td>
<td>0.05</td>
<td>0.01</td>
</tr>
</tbody>
</table>

calculate the **present value of this investment** under each scenario.
5. (2+4+4=10 points) A person is currently employed at age $x$ at time 0, which we call State 0. Let State 1 denote unemployment and State 2 denote deceased. The transition forces between states are as follows:

\[
\begin{align*}
\mu_{x+t}^{01} &= 0.20 + 0.0002t^2 & \mu_{x+t}^{02} &= \mu_{x+t}^{12} = 0.05 & \mu_{x+t}^{10} &= 0.80 - 0.04t & \mu^2 &= \mu^{21} = 0
\end{align*}
\]

a) Draw the transition state diagram for this model.

b) Obtain the Kolmogorov forward equation for $p_{00}^t$ and $p_{10}^t$ (or $p_{0}^{t0}$ and $p_{0}^{t1}$).

c) Using one-year time-steps ($h = 1$) to approximate the solutions to the Kolmogorov differential equations, estimate $p_{00}^t$ and $p_{10}^t$ (or $p_{0}^{t0}$ and $p_{0}^{t1}$) for $t = 1, 2, \ldots, 20$ by completing the missing values in the table below. (Be sure to provide some example calculations)

<table>
<thead>
<tr>
<th>$t$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{p_{00}^t}$</td>
<td>0.75</td>
<td>0.7144</td>
<td>0.6707</td>
<td>0.6288</td>
<td>0.5884</td>
<td>0.5121</td>
<td>0.4759</td>
<td>0.4408</td>
<td>0.4068</td>
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<tr>
<td>$t_{p_{01}^t}$</td>
<td>0.2</td>
<td>0.1882</td>
<td>0.1858</td>
<td>0.1853</td>
<td>0.1855</td>
<td>0.1862</td>
<td>0.1875</td>
<td>0.1894</td>
<td>0.1919</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>$t_{p_{00}^t}$</td>
<td>0.3737</td>
<td>0.3415</td>
<td>0.3099</td>
<td>0.2789</td>
<td>0.2484</td>
<td>0.2181</td>
<td>0.1879</td>
<td>0.1272</td>
<td>0.0962</td>
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<tr>
<td>$t_{p_{01}^t}$</td>
<td>0.1951</td>
<td>0.1989</td>
<td>0.2034</td>
<td>0.2149</td>
<td>0.2220</td>
<td>0.2302</td>
<td>0.2395</td>
<td>0.2501</td>
<td>0.2623</td>
<td></td>
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</tbody>
</table>
6. (3+3+2=8 points) A block of 1000 fully discrete insurances, issued at age 50, are in force at age 59. You are given:

1) The gross premium is $G = 16$
2) The ninth gross premium reserve is $9V = 115$
3) The tenth gross premium reserve is $10V = 128.83277$
4) The tenth year death benefit is 1000
5) The tenth year withdrawal benefit is 110.
6) The assumed interest rate is 6% per annum
7) Expenses are 3 per policy
8) There are no claim settlement expenses
9) Withdrawals can occur only at the end of the contract year
10) The assumed decrement rates are $q^{(d)}_{79} = 0.01$ and $q^{(w)}_{79} = 0.10$.

During the contract year, there are 13 deaths and 100 withdrawals. On this block of policies, calculate the following:

a) Gain from Mortality
b) Gain from Withdrawal
c) Total Gain

END OF TEST PAPER