

Instructions: Show Your Work!

1. (3 pts) Find the shortest distance between the spheres

$$x^2 + y^2 + z^2 = 4 \quad \text{and} \quad x^2 + y^2 + z^2 = 4x + 4y + 4z - 11.$$

2. (3 pts) Let $O(0, 0)$, $A(1, -1)$, $B(2, \alpha)$ and $C(\beta, 1)$ be points in \mathbb{R}^2 . Find values of α and β such that the vectors \vec{AB} and \vec{OC} are parallel.

3. (4 pts) Find the scalar and vector projections of \mathbf{v} onto \mathbf{u} where

$$\mathbf{u} = 3\mathbf{i} - 3\mathbf{j} + \mathbf{k}, \quad \mathbf{v} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$$

4. (2 pts) (**bonus**) The vector

$$\text{orth}_{\mathbf{a}}\mathbf{b} = \mathbf{b} - \text{proj}_{\mathbf{a}}\mathbf{b}$$

is called an **orthogonal projection** of \mathbf{b} . Show that $\text{orth}_{\mathbf{a}}\mathbf{b}$ is orthogonal to \mathbf{a}

Instructions: Show Your Work!

1. (3 pts) Find an equation of the set of all points whose distances from the point $A(0, 3, 0)$ are **double** their distances from the origin. Describe the set.
2. (3 pts) Let $O(0, 0)$, $A(1, -1)$, $B(2, \alpha)$ and $C(\beta, 1)$ be points in \mathbb{R}^2 . Find values of α and β such that the vectors \vec{AB} and \vec{OC} are parallel.

3. (4 pts) Find the scalar and vector projections of \mathbf{v} onto \mathbf{u} where

$$\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \mathbf{v} = 5\mathbf{i} - \mathbf{k}$$

4. (2 pts) (**bonus**) The vector

$$\text{orth}_{\mathbf{a}}\mathbf{b} = \mathbf{b} - \text{proj}_{\mathbf{a}}\mathbf{b}$$

is called an **orthogonal projection** of \mathbf{b} . Show that $\text{orth}_{\mathbf{a}}\mathbf{b}$ is orthogonal to \mathbf{a}