

Math201.15, Quiz # 2, Term 181

Name:

Soluhr

ID#:

Serial #:

1. [2 points] Find parametric equations for the line that passes through $(2, -1, 4)$ and is perpendicular to the plane $2x - y + 5z = 10$.

2. [3 points] Identify (name, center, axis) and sketch the surface

$$z^2 = \frac{x^2}{4} + y^2 - 2y.$$

3. [2.5 points] Let $f(x, y) = \frac{3}{\sqrt{y+x^2}}$.

a. Find and sketch the domain of f .

b. Find an equation for the level curve of f that passes through the point $(0, 1)$. Sketch the level curve.

4. [2.5 points] Find the limit if it exists: $\lim_{(x,y) \rightarrow (4,1)} \frac{xy+x-4y^2-4y}{x-2\sqrt{xy}}$.

Good luck,

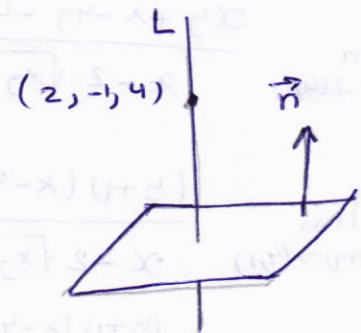
Ibrahim Al-Rasasi

1. a point on the line is $(2, -1, 4)$

• a vector parallel to the plane
= normal vector of the plane
= $\vec{n} = \langle 2, -1, 5 \rangle$

• parametric equations of the line are

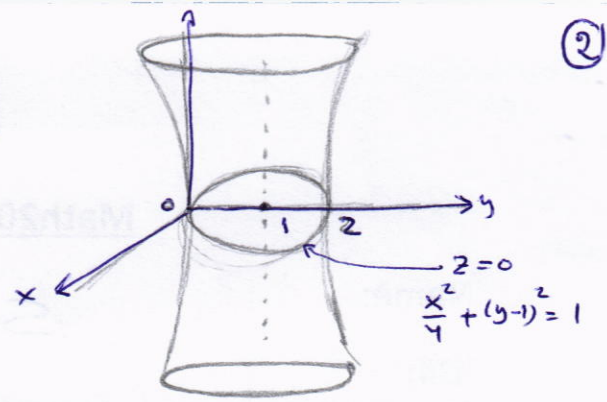
$$\begin{aligned} x &= 2 + 2t \\ y &= -1 - t \\ z &= 4 + 5t \end{aligned}, t \in \mathbb{R}$$



$$\textcircled{2} \quad z^2 = \frac{x^2}{4} + y^2 - 2y \Rightarrow z^2 = \frac{x^2}{4} + (y-1)^2 - 1$$

$$\Rightarrow \frac{x^2}{4} + (y-1)^2 - z^2 = 1 \quad \textcircled{1}$$

- a hyperboloid of one sheet $\textcircled{1}$
- center $(0, 1, 0)$
- axis: the line through $(0, 1, 0)$ and parallel to the z -axis.



$$\textcircled{3} \quad f(x, y) = \frac{3}{\sqrt{y+x^2}}$$

a) Domain = $\{(x, y) \in \mathbb{R}^2 : y + x^2 > 0\} = \{(x, y) \in \mathbb{R}^2 : y > -x^2\}$

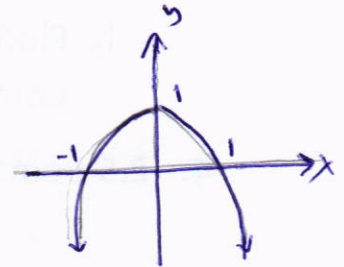
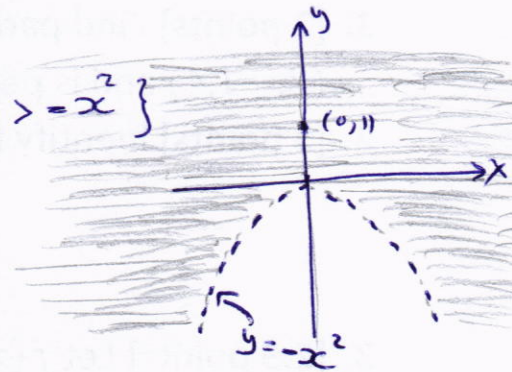
b) An equation for the level curve is

$$f(x, y) = c \Rightarrow \frac{3}{\sqrt{y+x^2}} = c$$

To find c , substitute $(0, 1)$:

$$\frac{3}{\sqrt{1+0}} = c \Rightarrow c = 3$$

The equation is $\frac{3}{\sqrt{y+x^2}} = 3 \Rightarrow y+x^2 = 1 \Rightarrow y = -x^2 + 1$.



$\textcircled{4}$

$$\lim_{(x, y) \rightarrow (4, 1)} \frac{xy + x - 4y^2 - 4y}{x - 2\sqrt{xy}}$$

Factor:

$$x(y+1) - 4y(y+1) = (y+1)(x-4y) \quad \textcircled{1}$$

$$= \lim_{(x, y) \rightarrow (4, 1)} \frac{(y+1)(x-4y)}{x-2\sqrt{xy}} \cdot \frac{x+2\sqrt{xy}}{x+2\sqrt{xy}}$$

$$= \lim_{(x, y) \rightarrow (4, 1)} \frac{(y+1)(x-4y)(x+2\sqrt{xy})}{x^2 - 4xy}$$

$$= \lim_{(x, y) \rightarrow (4, 1)} \frac{(y+1)(x-4y)(x+2\sqrt{xy})}{x(x-4y)}$$

$$= \lim_{(x, y) \rightarrow (4, 1)} \frac{(y+1)(x+2\sqrt{xy})}{x}$$

$$= \frac{2(4+4)}{4} = 4$$

$\frac{0.5}{0.5}$

Math201.10, Quiz # 2, Term 181

Name:

Solutions

ID#:

Serial #:

1. [2.5 points] Find the point of intersection, if it exists, between the following two lines:

$$L_1: x = 2 - t, \quad y = 1 - 3t, \quad z = 5 + 5t$$

$$L_2: x = 2s, \quad y = 4 + s, \quad z = -12 + 5s.$$

2. [2.5 points] Identify (name, vertex, axis) and sketch the surface

$$z^2 + 1 = 2x^2 + 3y^2 - 6y + 4.$$

3. [2.5 points] Let $f(x, y) = \ln(x^3 - y)$.

a. Find and sketch the domain of f .

b. Find an equation for the level curve of f that passes through the point $(1, -1)$. Sketch the level curve.

4. [2.5 points] Find the limit if it exists: $\lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^3}{x^3+y^3}$.

Good luck,

Ibrahim Al-Rasasi

$$\begin{aligned} x=x &\Rightarrow 2-t=2s \\ y=y &\Rightarrow 1-3t=4+s \\ z=z &\Rightarrow 5+5t=-12+5s \end{aligned} \Rightarrow \begin{cases} -t-2s=-2 & (1) \\ -3t-s=3 & (2) \\ 5t-5s=-17 & (3) \end{cases} \Rightarrow \begin{aligned} &3t+6s=6 \\ &-3t-s=3 \quad \text{Add} \\ \hline &5s=9 \\ &s=\frac{9}{5} \end{aligned}$$

$$\begin{aligned} (1) \Rightarrow t &= 2 - 2\left(\frac{9}{5}\right) \\ &= 2 - \frac{18}{5} = \frac{-8}{5} \\ &t = -\frac{8}{5} \end{aligned}$$

Sub. $t = -\frac{8}{5}$ & $s = \frac{9}{5}$ in (3): $-\frac{40}{5} + \frac{45}{5} = -8 + 9 = 1 \neq -17$ ✓

So the solution of the system is $s = \frac{9}{5}, t = -\frac{8}{5}$

Sub. $t = -\frac{8}{5}$ in L_1 : $x = 2 + \frac{8}{5} = \frac{18}{5}$; $y = 1 + \frac{24}{5} = \frac{29}{5}$; $z = 5 - 8 = -3$

The point of intersection is $(\frac{18}{5}, \frac{29}{5}, -3)$.