MATH 202.5 (Term 181)

Quiz 3 (Sects. 4.1.3, 4.2 & 4.3) Duration: 20min

Name: ID number:

1.)(3pts) Let L be a linear differential operator. If $Ly_{p_1} = \cos(\frac{x}{2})$ and $Ly_{p_2} = \sin(\frac{x}{2})$, then find a particular solution of the DE $Ly = 4\cos^2(\frac{x}{4}) - 2 + \sin(\frac{x}{4})\cos(\frac{x}{4})$.

2.)(4pts)Use reduction of order to find a second solution y_2 of the DE: $(1-x^2)y'' + 2xy' - 2y = 0$, giving that $y_1 = 1 + x^2$ is a solution.

3.)(3pts)Solve the DE: $y''' + y' - 2y = 0$

1.) $\cos\frac{x}{2} = 2\cos^2(\frac{x}{4}) - 1$

$\sin(\frac{x}{2}) = 2\sin(\frac{x}{4})\cos(\frac{x}{4})$

$4\cos^2\frac{x}{4} - 2 = 2\cos(\frac{x}{2})$

$\sin(\frac{x}{4})\cos(\frac{x}{4}) = \frac{1}{2}\sin(\frac{x}{2})$

$4\cos^2(\frac{x}{4}) - 2 + \sin\frac{x}{4}\cos\frac{x}{4} = 2\cos\frac{x}{2} + \frac{1}{2}\sin\frac{x}{2}$

$\Rightarrow y_p = 2y_{p_1} + \frac{1}{2}y_{p_2}$

2.) $y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} dx$

$P(x) = \frac{2x}{1-x^2}$, $-\int P(x)\,dx = \int \frac{-2x}{1-x^2}dx$

$= \ln|1-x^2|$

$y_2 = (1+x^2)\int \frac{1-x^2}{(1+x^2)^2}dx$, $x \in (-1,1)$

$\frac{1+x^2}{(1+x^2)^2} - \frac{2x^2}{(1+x^2)^2}$

$\frac{1}{1+x^2} - \frac{2x^2}{(1+x^2)^2}$

$\underbrace{\int \frac{-2x^2}{(1+x^2)^2} = \frac{x}{1+x^2} - \int \frac{dx}{1+x^2}}_{\text{by integration by parts}}$

$\Rightarrow y_2 = (1+x^2)\left[\tan^{-1}x + \frac{x}{1+x^2} - \tan^{-1}x\right]$

$= x$

3.) The auxiliary equation

$m^3 + m - 2 = 0$

$(m-1)(m^2+m+2) = 0$

$m = 1$, $m = \frac{-1}{2} \pm i\frac{\sqrt{7}}{2}$

$y = c_1 e^{x} + c_2 e^{-\frac{x}{2}}\cos\frac{\sqrt{7}}{2}x + c_3 e^{-\frac{x}{2}}\sin\frac{\sqrt{7}}{2}x$

end

MATH 202.6 (Term 181)

Quiz 3 (Sects. 4.1.3, 4.2 & 4.3) Duration: 20min

Name: ID number:

1.)(3pts) Let L be a linear differential operator. If $Ly_{p_1} = \cos(4x)$ and $Ly_{p_2} = \sin(4x)$, then find a particular solution of the DE $Ly = \cos^2(2x) - \frac{1}{2} + 4\sin(2x)\cos(2x)$.

2.)(4pts)Use reduction of order to find a second solution y_2 of the DE: $(1-x^2)y'' + 2xy' - 2y = 0$, giving that $y_1 = 1 + x^2$ is a solution.

3.)(3pts)Solve the DE: $y''' + y = 0$.

---

1) $\cos(4x) = 2\cos^2(2x) - 1$

$\sin(4x) = 2\sin(2x)\cos(2x)$

$\Rightarrow \cos^2(2x) - \frac{1}{2} = \frac{1}{2}\cos 4x$

$4\sin 2x \cos 2x = 2\sin 4x$

$\cos^2 2x - \frac{1}{2} + 4\sin 2x\cos 2x = \frac{1}{2}\cos 4x + 2\sin 4x$

$\Rightarrow y_p = \frac{1}{2}y_{p_1} + 2y_{p_2}$

2) $y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} dx.$

$P(x) = \frac{2x}{1-x^2}$, $-\int P(x)dx = \int \frac{-2x}{1-x^2}dx$

$= \ln|1-x^2|$

$$y_2 = (1+x^2)\int \underbrace{\frac{1-x^2}{(1+x^2)^2}}_{\frac{1+x^2}{(1+x^2)^2} - \frac{2x^2}{(1+x^2)^2}} dx, \quad x \in (-1,1)$$

$\frac{1}{1+x^2} - \frac{2x^2}{(1+x^2)^2}$

$$\left.\begin{aligned}\int \frac{-2x^2}{(1+x^2)^2} &= \frac{x}{1+x^2} - \int \frac{dx}{1+x^2} \\ &= \frac{x}{1+x^2} - \tan^{-1}x\end{aligned}\right\} \text{By integration by parts}$$

$\Rightarrow y_2 = (1+x^2)\left[\tan^{-1}x + \frac{x}{1+x^2} - \tan^{-1}x\right]$

$= x$

3) The auxiliary equation is

$m^3 + 1 = 0$

$(m+1)(m^2 - m + 1) = 0$

$m = -1$, $m = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

$y = c_1 e^{-x} + c_2 e^{\frac{x}{2}}\cos\frac{\sqrt{3}}{2}x + c_3 e^{\frac{x}{2}}\sin\frac{\sqrt{3}}{2}x$

End