1.) (3pts) Let $L$ be a linear differential operator. If $Ly_1 = \cos\left(\frac{x}{2}\right)$ and $Ly_2 = \sin\left(\frac{x}{2}\right)$, then find a particular solution of the DE $Ly = 4\cos^2\left(\frac{x}{4}\right) - 2 + \sin\left(\frac{x}{4}\right)\cos\left(\frac{x}{4}\right)$.

2.) (4pts) Use reduction of order to find a second solution $y_2$ of the DE:

$$(1 - x^2)y'' + 2xy' - 2y = 0,$$

given that $y_1 = 1 + x^2$ is a solution.

3.) (3pts) Solve the DE: $y'' + y' - 2y = 0$.

\[
\int \frac{-2x^2}{(1 + x^2)^2} = \frac{x}{1 + x^2} - \int \frac{dx}{1 + x^2} \\
\] 

by integration by parts

\[
y_2 = (1 + x^2)\left[\tan^{-1}x + \frac{x}{1 + x^2} - \tan^{-1}1\right] = x
\]

3.) The auxiliary equation $m^2 + m - 2 = 0$ gives $m = 1, -2$.

\[
y = c_1 e^{x} + c_2 e^{-2x}\cos\frac{\sqrt{5}}{2}x + c_3 e^{-2x}\sin\frac{\sqrt{5}}{2}x
\]
1.) (3pts) Let \( L \) be a linear differential operator. If \( Ly_{p_1} = \cos(4x) \) and \( Ly_{p_2} = \sin(4x) \), then find a particular solution of the DE \( Ly = \cos^2(2x) - \frac{1}{2} + 4 \sin(2x) \cos(2x) \).

2.) (4pts) Use reduction of order to find a second solution \( y_2 \) of the DE: 
\[
(1 - x^2)y'' + 2xy' - 2y = 0,
\]
giving that \( y_1 = 1 + x^2 \) is a solution.

3.) (3pts) Solve the DE: 
\[
y'' + y = 0.
\]