MATH 202.5 (Term 181)

Quiz 5 (Sects. 6.2 & 6.3) Duration: 30min

Name: ID number:

1.)(5pts)Find 2 powers series solutions of the DE: $(x+1)y''+y'+y=0$.
2.)(2pts)Find the indicial roots of the DE $x^2y''+x(x^4-1)y'-(1+x^3)y=0$ at $x=0$.
3.)(3pts)Find a relation of recurrence satisfied by c_n, where $y=\sum_{n=0}^{\infty}c_nx^{n+\frac{1}{2}}$ is solution of the DE $2xy''+y'-y=0$.

Solution

1.) $y=\sum_{n=0}^{\infty}c_nx^n \quad , |x|<1$

$(x+1)\sum_{n=2}^{\infty}n(n-1)c_nx^{n-2}+\sum_{n=1}^{\infty}nc_nx^{n-1}+\sum_{n=0}^{\infty}c_nx^n=0$

$2c_2+c_0+c_1+\sum_{k=1}^{\infty}\left[(k+1)(k+2)c_{k+2}+(k+1)^2c_{k+1}+c_k\right]x^k=0$

$c_2=-\frac{1}{2}(c_0+c_1)$

$c_{k+2}=-\frac{(k+1)^2c_{k+1}+c_k}{(k+1)(k+2)}, \quad k=1,2,3,..$

- $c_0\neq0,\ c_1=0$

$c_2=-\frac{c_0}{2},\ c_3=\frac{c_0}{6}$

$y=c_0\underbrace{\left(1-\frac{x^2}{2}+\frac{x^3}{6}+\cdots\right)}_{y_1}$

- $c_0=0,\ c_1\neq0$

$c_2=-\frac{c_1}{2},\ c_3=\frac{c_1}{12}$

$y=c_1\underbrace{\left(x-\frac{x^2}{2}+\frac{x^3}{12}+\cdots\right)}_{y_2}$

$y=c_1y_1+c_2y_2$

2) $p(x)=\frac{x^4-1}{1} \Rightarrow p_0=-1$

$q(x)=-(1+x^3) \Rightarrow q_0=-1$

$r(r-1)-r-1=0$

$r=1\pm\sqrt{2}$

3.) $\sum_{n=0}^{\infty}2c_n(n+\frac{1}{2})(n-\frac{1}{2})x^{n-\frac{1}{2}}+\sum_{n=0}^{\infty}c_n(n+\frac{1}{2})x^{n-\frac{1}{2}}-\sum_{n=0}^{\infty}c_nx^{n+\frac{1}{2}}=0$

$x^{\frac{1}{2}}\left(\sum 2c_n(n+\frac{1}{2})(n-\frac{1}{2})x^{n-1}+\sum_{n=1}^{\infty}c_n(n+\frac{1}{2})x^{n-1}-\sum_{n=0}^{\infty}c_nx^n=0\right)$

$\sum_{k=0}^{\infty}\left[(k+1)(2k+3)c_{k+1}-c_k\right]x^k=0$

$c_{k+1}=\frac{c_k}{(k+1)(2k+3)}, \quad k=0,1,2,\ldots$

$c_1=\frac{c_0}{3},\quad c_2=\frac{c_0}{30},\quad c_3=\cdots$

$y=c_0\underbrace{\left(1+\frac{x}{3}+\frac{x^2}{30}+\cdots\right)}_{y_1}$

MATH 202.6 (Term 181)

Quiz 5 (Sects. 6.2 & 6.3) Duration: 30min

Name: ID number:

1.)(5pts)Find 2 powers series solutions of the DE: $(x-1)y''-y'-y=0$.
2.)(2pts)Find the indicial roots of the DE $2x^2y''-x(2+x^3)y'-(1+x^5)y=0$ at $x=0$.
3.)(3pts)Find a relation of recurrence satisfied by c_n, where $y=\sum_{n=0}^{\infty} c_n x^{n+\frac{1}{3}}$ is solution of the DE $3xy''+2y'-y=0$.

Solution

1.) $y=\sum_{n=0}^{\infty} c_n x^n$, $|x|<1$

$$(x-1)\sum_{n=2}^{\infty} n(n-1)x^{n-2}-\sum_{n=1}^{\infty} nc_n x^{n-1}-\sum_{n=0}^{\infty} c_n x^n=0$$

$$-2c_2-c_1-c_0+\sum_{k=1}^{\infty}\left[(k+1)(k-1)c_{k+1}-(k+1)(k+2)c_{k+2}-c_k\right]x^k=0$$

$$c_2=-\frac{1}{2}(c_0+c_1)$$

$$c_{k+2}=\frac{(k+1)(k-1)c_{k+1}-c_k}{(k+1)(k+2)}, k=1,2,$$

- $c_0\neq 0$, $c_1=0$

$\Rightarrow c_2=-\frac{c_0}{2}$, $c_3=0$, $c_4=\frac{c_0}{24}$, $c_5=$

$$y=c_0\underbrace{\left(1-\frac{x^2}{2}+\frac{x^4}{24}+\cdots\right)}_{y_1}$$

- $c_0=0$, $c_1\neq 0$

$\Rightarrow c_2=-\frac{c_1}{2}$, $c_3=\frac{c_1}{6}$, $c_4=\frac{c_1}{72}$

$$y=c_1\underbrace{\left(x-\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{72}+\cdots\right)}_{y_2}$$

$$\underline{y=c_1y_1+c_2y_2}$$

2.) $p(x)=\frac{-(2+x^3)}{2}\Rightarrow p_0=-1$

$q(x)=\frac{-(1+x^5)}{2}\Rightarrow q_0=-\frac{1}{2}$

$r(r-1)-r-\frac{1}{2}=0$

$$\underline{r=2\pm\sqrt{6}}$$

3.) $$\sum_{n=0}^{\infty}3(n+\tfrac{1}{3})(n-\tfrac{2}{3})c_n x^{n-\frac{2}{3}}+\sum_{n=0}^{\infty}2c_n(n+\tfrac{1}{3})x^{n-\frac{2}{3}}-\sum_{n=0}^{\infty}c_n x^{n+\frac{1}{3}}=0$$

$$x^{\frac{1}{3}}\left(\sum_{n=1}^{\infty}3c_n(n+\tfrac{1}{3})(n-\tfrac{2}{3})x^{n-1}+\sum_{n=1}^{\infty}2c_n(n+\tfrac{1}{3})x^{n-1}-\sum_{n=0}^{\infty}c_n x^n\right)=0$$

$$\sum_{k=0}^{\infty}\left[c_{k+1}(3k+4)(k+1)-c_k\right]x^k=0$$

$$c_{k+1}=\frac{c_k}{(k+1)(3k+4)}, k=0,1,2..$$

$c_1=\frac{c_0}{4}$, $c_2=\frac{c_0}{56}$, $c_3=$

$$y=c_0\underbrace{\left(1+\frac{x}{4}+\frac{x^2}{56}+\cdots\right)}_{y_1}$$