MATH 202.5 (Term 181)

Quiz 6 (Sects. 8.2 & 8.3) Duration: 30min

Name: **ID number:**

1.)(5pts)Solve the IVP $X' = \begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix} X$, $X(0) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

2.)(5pts)Solve the IVP $X' = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} X$, $X(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

3.)(5pts)Solve the system $X' = AX + \begin{pmatrix} 3t \\ e^{-t} \end{pmatrix}$, **given that** $\Phi(t) = \begin{pmatrix} e^{-t} & -2e^{2t} \\ e^{-t} & e^{2t} \end{pmatrix}$ **is a fundamental matrix of** $X' = AX$.

1.) $(4-\lambda)(2-\lambda)+1=0$, $\lambda = 3,3$

$(A-3I)K=0$, $\left(\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & -1 & 0 \end{array}\right)$ $x+y=0$ $K\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$(A-3I)P=K$, $\left(\begin{array}{cc|c} 1 & 1 & 1 \\ -1 & -1 & -1 \end{array}\right)$ $x+y=1$ $P\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{3t}$

$X_2 = \left[t\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] e^{3t} = \begin{pmatrix} t \\ -t+1 \end{pmatrix} e^{3t}$

$X = c_1 X_1 + c_2 X_2$

$X(0) = c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} c_1 = 0 \\ c_2 = -1 \end{matrix}$

2.) $(3-\lambda)(2-\lambda)+1=0$, $\lambda = \frac{5}{2} \pm i\frac{\sqrt{3}}{2}$

$\left(A - \left(\frac{5}{2} + i\frac{\sqrt{3}}{2}\right)I\right)K = 0$

$\left(\begin{array}{cc|c} \frac{1}{2} - i\frac{\sqrt{3}}{2} & -1 & 0 \\ 1 & -\frac{1}{2} - i\frac{\sqrt{3}}{2} & 0 \end{array}\right)$ $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)x - y = 0$ $K\begin{pmatrix} 2 \\ 1 - i\sqrt{3} \end{pmatrix}$

$X_1 = \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix}\cos\frac{\sqrt{3}}{2}t - \begin{pmatrix} 0 \\ -\sqrt{3} \end{pmatrix}\sin\frac{\sqrt{3}}{2}t\right] e^{\frac{5}{2}t}$

$X_2 = \left[\begin{pmatrix} 0 \\ -\sqrt{3} \end{pmatrix}\cos\frac{\sqrt{3}}{2}t + \begin{pmatrix} 2 \\ 1 \end{pmatrix}\sin\frac{\sqrt{3}}{2}t\right] e^{\frac{5}{2}t}$

$X = c_1 X_1 + c_2 X_2$

$X(0) = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2\begin{pmatrix} 0 \\ -\sqrt{3} \end{pmatrix} \Rightarrow \begin{matrix} c_1 = -1/2 \\ c_2 = -\sqrt{3}/2 \end{matrix}$

3.) $\Phi^{-1} = \frac{1}{3e^{t}}\begin{pmatrix} e^{2t} & 2e^{2t} \\ -e^{-t} & e^{-t} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} e^{t} & 2e^{t} \\ -e^{-2t} & e^{-2t} \end{pmatrix}$

$\Phi^{-1}F = \frac{1}{3}\begin{pmatrix} 3te^{t} + 2 \\ -3te^{-2t} + e^{-3t} \end{pmatrix}$

$\int \Phi^{-1}F = \frac{1}{3}\begin{pmatrix} 3(t-1)e^{t} + 2t \\ \left(\frac{3}{2}t + \frac{3}{4}\right)e^{-2t} - \frac{1}{3}e^{-3t} \end{pmatrix}$

$\underbrace{\Phi\int\Phi^{-1}F}_{X_p} = \frac{1}{3}\begin{pmatrix} -\frac{9}{2} + \left(2t + \frac{2}{3}\right)e^{-t} \\ \frac{9t}{2} - \frac{9}{4} + \left(2t - \frac{1}{3}\right)e^{-t} \end{pmatrix}$

$X = \Phi C + X_p$

MATH 202.6 (Term 181)

Quiz 6 (Sects. 8.2 & 8.3) **Duration: 30min**

Name: **ID number:**

1.)(5pts)Solve the IVP $X' = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} X$, $X(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

2.)(5pts)Solve the IVP $X' = \begin{pmatrix} 4 & -1 \\ 1 & 3 \end{pmatrix} X$, $X(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

3.)(5pts)Solve the system $X' = AX + \begin{pmatrix} e^t \\ 2t \end{pmatrix}$, **given that** $\Phi(t) = \begin{pmatrix} e^{-2t} & 2e^t \\ e^{-2t} & 3e^t \end{pmatrix}$ **is a fundamental matrix of** $X' = AX$.

1.) $(3-\lambda)(1-\lambda)+1=0, \quad \lambda = 2, 2$

$(A-2I)K=0, \quad \left(\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & -1 & 0 \end{array}\right) \quad x+y=0 \quad K\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$(A-2I)P=K, \quad \left(\begin{array}{cc|c} 1 & 1 & 1 \\ -1 & -1 & -1 \end{array}\right) \quad x+y=1 \quad P\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t}$

$X_2 = \left[t\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] e^{2t} = \begin{pmatrix} t \\ -t+1 \end{pmatrix} e^{2t}$

$X = c_1 X_1 + c_2 X_2$

$X(0) = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} c_1 = 1 \\ c_2 = 0 \end{matrix}$

2.) $(4-\lambda)(3-\lambda)+1=0, \quad \lambda = \frac{7}{2} \pm i\frac{\sqrt{3}}{2}$

$\left(A - \left(\frac{7}{2} + i\frac{\sqrt{3}}{2}\right)I\right)K = 0$

$\left(\begin{array}{cc|c} \frac{1}{2} - i\frac{\sqrt{3}}{2} & -1 & 0 \\ 1 & -\frac{1}{2} - i\frac{\sqrt{3}}{2} & 0 \end{array}\right) \quad \left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)x - y = 0$

$K\begin{pmatrix} 1 \\ \frac{1}{2} - i\frac{\sqrt{3}}{2} \end{pmatrix}$

$X_1 = \left[\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \cos\frac{\sqrt{3}}{2}t - \begin{pmatrix} 0 \\ -\frac{\sqrt{3}}{2} \end{pmatrix} \sin\frac{\sqrt{3}}{2}t\right] e^{\frac{7}{2}t}$

$X_2 = \left[\begin{pmatrix} 0 \\ -\frac{\sqrt{3}}{2} \end{pmatrix} \cos\frac{\sqrt{3}}{2}t + \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \sin\frac{\sqrt{3}}{2}t\right] e^{\frac{7}{2}t}$

$X = c_1 X_1 + c_2 X_2$

$X(0) = c_1 \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ -\frac{\sqrt{3}}{2} \end{pmatrix} \Rightarrow \begin{matrix} c_1 = 1 \\ c_2 = \frac{\sqrt{3}}{3} \end{matrix}$

3.) $\Phi^{-1} = \frac{1}{e^{-t}} \begin{pmatrix} 3e^t & -2e^t \\ -e^{-2t} & e^{-2t} \end{pmatrix} = \begin{pmatrix} 3e^{2t} & -2e^{2t} \\ -e^{-t} & e^{-t} \end{pmatrix}$

$\Phi^{-1}F = \begin{pmatrix} 3e^{3t} - 4te^{2t} \\ -1 + 2te^{-t} \end{pmatrix}$

$\int \Phi^{-1}F = \begin{pmatrix} e^{3t} - (2t-1)e^{2t} \\ -t - 2(t+1)e^{-t} \end{pmatrix}$

$\Phi \int \Phi^{-1} F = \begin{pmatrix} (1-2t)e^t \cancel{+6t+3} \\ (1-3t)e^t - 8t - 5 \end{pmatrix}$

X_p

$X = \Phi C + X_p$