Name:________________________  ID Number:________________________

Section Number:______________  Serial Number:________________________

Class Time:___________________  Instructor’s Name:____________________

Instructions:

1. Calculators and Mobile Phones not allowed.
2. Please write legibly.
3. Make sure that you have eight pages of problems (Total of eight Problems).

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<tr>
<th>Question Number</th>
<th>Points</th>
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1. (10 Points) Find the rank of the following matrix

\[
A = \begin{bmatrix}
2 & -1 & 1 & 0 \\
3 & 2 & 0 & 1 \\
-1 & 4 & -2 & 1 \\
\end{bmatrix}
\]
2. (12 Points) Find a basis and the dimension of the subspace

\[ U = \{(x_1, x_2, x_3, x_4) \mid x_1 + x_2 = x_3 + x_4\}. \]
3. (16 Points) Find the general solution of the equation $y'' - 2y' + y = \frac{e^x}{x}, \ (x > 0)$. 
4. (12 Points) Find the general solution of the equation $y'' + 4y = 5 \sin 2x + 2xe^x + 1$. Do not evaluate the constants in the particular solution.
5. (14 Points) [This question has three parts: a, b, and c] 
The three solutions of the third order differential equation $y''' - y'' + y' - y = 0$ are given by: $y_1 = e^x$, $y_2 = \cos x$, $y_3 = \sin x$.

(a) Verify that the three solutions are linearly independent.

(b) Write the general solution of the equation.
(c) Find a particular solution if $y(0) = 1$, $y'(0) = 2$, and $y''(0) = -1$. 
6. (10 Points) The general solution of a constant coefficient differential equation is: 
\[ y = (c_1 + c_2 x)e^x + c_3 \cos(2x) + c_4 \sin(2x). \] Construct the corresponding constant coefficient differential equation.
7. (10 Points) Transform the following differential equation into an equivalent system of first-order differential equations, and write the system in matrix form: 

\[ x''' - (\sin t)x'' + e^t x = t^2. \]
8. (16 Points) [This question has two parts: a and b] Consider the homogeneous system of first order differential equations:

\[
\begin{bmatrix}
  x' \\
  y' \\
  z'
\end{bmatrix} =
\begin{bmatrix}
  4 & 1 & 4 \\
  1 & 7 & 1 \\
  4 & 1 & 4
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
\]

The three solutions of this system are:

\[
X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{9t}, \quad X_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} e^{6t}, \quad X_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.
\]

(a) Verify that \(X_1\) is a solution.
(b) Find a particular solution satisfying $x(0) = 0$, $y(0) = 1$, $z(0) = 1$. 