Exercise 1 [10 pt]. For each of the following sets, decide whether it is a subspace of $\mathbb{R}^3$ or not.

\[
E_1 = \{(x, y, z) \in \mathbb{R}^3 \mid 3x - 7y = z\}
\]
\[
E_2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 - z^2 = 0\}
\]
\[
E_3 = \{(x, y, z) \in \mathbb{R}^3 \mid x + y - z = x + y + z = 0\}
\]
\[
E_4 = \{(x, y, z) \in \mathbb{R}^3 \mid z(x^2 + y^2) = 0\}
\]
Exercise 2[10 pt]. Find all values of $t$ such that

$$\{(1, 0, t), (1, 1, t), (t, 0, 1)\}$$

is a basis of $\mathbb{R}^3$
Exercise 3[10 pt]. Consider the set

\[ E = \mathbb{R}_+^* \times \mathbb{R} \]

equipped with the operations:

\[ + : (a, b) + (a', b') = (aa', b + b') \]

\[ (\forall \lambda \in \mathbb{R}) (\forall (a, b) \in E) \lambda (a, b) = (a^\lambda, \lambda b). \]

Show that \((E, +, \cdot)\) is a real vector space.
Exercise 4[10 pt].

- Show that

\[ S = \{ \mathbf{e}_1 = (1, 1, 1), \mathbf{e}_2 = (1, 1, 2), \mathbf{e}_3 = (1, 2, 3) \} \]

is a basis of \( \mathbb{R}^3 \).

- For each vector \( \mathbf{u} = (a, b, c) \) of \( \mathbb{R}^3 \), find the vector coordinates of \( \mathbf{u} \) with respect to the basis \( S \).
Exercise 5[10 pt]. Find a basis and the dimension of the following real vector space

\[ V = \{ (x, y, z, t) \in \mathbb{R}^4 \mid x - 2y + 3z + t = 0 \}. \]
Exercise 6[10 pt]. Let $B = (v_1, v_2, v_3)$ and $B' = (u_1, u_2, u_3)$, where

\[ v_1 = (1,1,1), v_2 = (2,3,2), v_3 = (1,5,4), u_1 = (1,1,0), u_2 = (1,2,0), u_3 = (1,2,1). \]

1. Show that $B$ and $B'$ are basis of $\mathbb{R}^3$.
2. Find the transition matrix from $B$ to $B'$.
3. Let $x = av_1 + bv_2 + cv_3$, find the coordinates of $x$ with respect to the basis $B'$. 