MATH 102 – Correction of Quiz1 (Term 182) **Section** 13

Exercise 1(5 points)

Let A be the area of the region that lies under the graph of $f(x) = e^{-x}$ between $x = 0$ and $x = 2$.

a) Using right endpoints find an expression of A as a limit. Do not evaluate the limit.

b) Estimate the area by taking the sample points to be midpoints and using four subintervals.

Solution

a) (2,5 pts)The expression for the area A of a function is given by

$A = \lim_{n\to+\infty} \sum_{i=1}^{i=n} f(x_i)\Delta x$ where $\Delta x = \frac{b-a}{n}$ and $x_i = x_0 + i\Delta x$. In our case $\Delta x = \frac{2-0}{n}$ and $x_i = \frac{2i}{n}$

Then $A = \lim_{n\to+\infty} \sum_{i=1}^{i=n} e^{-\frac{2i}{n}} \frac{2}{n}$

b) (2,5 pts) The midpoints of the four intervals are $x_1^* = \frac{1}{4}$, $x_2^* = \frac{3}{4}$, $x_3^* = \frac{5}{4}$ and $x_4^* = \frac{7}{4}$

Then an estimation of the area is given by:

$$A = \frac{1}{2}\left(e^{-\frac{1}{4}} + e^{-\frac{3}{4}} + e^{-\frac{5}{4}} + e^{-\frac{7}{4}}\right)$$

Exercice 2 (5 points)

1) Express the limit as an integral on the given interval $\sum_{i=1}^{n} x_i \ln(1+x_i^2)\Delta x$, on $[2,6]$

2) Use the definition of the integral as limit of Riemann sums to evaluate the integral $\int_{-1}^{5}(1+3x)dx$

Solution

1) (2,5 pts) It is clear that the limit under consideration is of the form

$$A = \lim_{n\to+\infty}\sum_{i=1}^{i=n} f(x_i)\Delta x = \int_a^b f(x)dx$$

then $\lim_{n\to+\infty}\sum_{i=1}^{n} x_i \ln(1+x_i^2)\Delta x = \int_2^6 x\ln(1+x^2)dx$

2) (2,5 pts) We know that $\int_{-1}^{5}(1+3x)dx$ is the limit of Riemann sums that is

$$\int_{-1}^{5}(1+3x)dx = \lim_{n\to+\infty}\sum_{i=1}^{n}(1+3x_i)\Delta x, \text{ where } \Delta x = \frac{5-(-1)}{n} = \frac{6}{n} \text{ and } x_i = -1+i\frac{6}{n}$$

$$\sum_{i=1}^{n}(1+3x_i)\Delta x = \frac{6}{n}\sum_{i=1}^{n}(1+3\left(-1+i\frac{6}{n}\right)) = \frac{6}{n}\sum_{i=1}^{n}(-2+i\frac{18}{n}) = \frac{6}{n}\left[-2n+\frac{18}{n}\frac{n(n+1)}{2}\right]$$

Then

$$\int_{-1}^{5}(1+3x)dx = \lim_{n\to+\infty}\frac{6}{n}\left[-2n+\frac{18}{n}\frac{n(n+1)}{2}\right] = -12+54 = 42,$$

because $\lim_{n\to+\infty}\frac{n(n+1)}{n^2} = 1$.