MATH 102 – Quiz 4 (Term 182) **Section** 14

Name: ID #:

Exercise 1 (5 points)

Which of the following sequences converge and which diverge? Find the limit of each convergent sequence.

a) $a_n = 2 + (0,1)^n$ b) $b_n = \dfrac{n + (-1)^n}{n}$ c) $c_n = \dfrac{n^2 - 2n + 1}{n - 1}$ d) $d_n = \dfrac{\sin(n)}{n}$

Solution

a) It is clear that $0,1 < 1$, then $\lim_{n\to+\infty}(0,1)^n = 0$. Therefore $\lim_{n\to+\infty} 2 + (0,1)^n = 2$.

b) $b_n = \dfrac{n + (-1)^n}{n} = 1 + \dfrac{(-1)^n}{n}$ and $\dfrac{-1}{n} \leq \dfrac{(-1)^n}{n} \leq +\dfrac{1}{n}$

It is obvious that the sequences $\left(\dfrac{-1}{n}\right)$ and $\left(\dfrac{1}{n}\right)$ converge to 0. Therefore the sequence $\left(\dfrac{(-1)^n}{n}\right)$ converges to 0 andf we conclude that the sequence (b_n) converges to 1.

c) $\lim_{n\to+\infty} \dfrac{n^2 - 2n + 1}{n - 1} = \lim_{n\to+\infty} \dfrac{n^2}{n} = \lim_{n\to+\infty} n = +\infty$, then (c_n) is divergent.

d) We know that $\dfrac{-1}{n} \leq \dfrac{\sin(n)}{n} \leq +\dfrac{1}{n}$ and that $\left(\dfrac{-1}{n}\right)$ and $\left(\dfrac{1}{n}\right)$ converge to 0, then $\left(\dfrac{\sin(n)}{n}\right)$ converges to 0.

Exercise 2 (5 points)

Consider the following series $\sum_{n=1}^{+\infty} 2^{2n} 3^{1-n}$.

1) Write this series as a geometric series. Is this series convergent or divergent?

2) Show that the series $\sum_{n=1}^{+\infty} \frac{1}{n(n+1)}$ is convergent and evaluate its sum.

Solution

1) We can write $2^{2n}3^{1-n} = \frac{1}{3}\left(\frac{2^2}{3}\right)^n = \frac{1}{3}\left(\frac{4}{3}\right)^n$

Then the series in question may be written as a geometric series as follows

$$\sum_{n=1}^{+\infty} 2^{2n}3^{1-n} = \sum_{n=1}^{+\infty} \frac{1}{3}\left(\frac{4}{3}\right)^n$$

with ratio $\frac{4}{3} > 1$. Then this series is divergent.

2) Let S_n the partial sum of order n, $S_n = \sum_{k=1}^{n} \frac{1}{k(k+1)}$

Notice that we can write $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$. Then

$S_n = \sum_{k=1}^{n} \frac{1}{k(k+1)} = S_n = \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1}\right) = \left(1 + \frac{1}{2} + ... + \frac{1}{n}\right) - \left(\frac{1}{2} + ... + \frac{1}{n+1}\right) = \left(1 - \frac{1}{n+1}\right)$

Then $\lim_{n\to+\infty} S_n = \lim_{n\to+\infty} \left(1 - \frac{1}{n+1}\right) = 1$.

Therefore the series is convergent and its sum is equal to 1.