1. The function \( f(x) = 4x^3 - 10x^2 - 8x + 3 \) is decreasing on

(a) \((-\frac{1}{3}, 2)\)
(b) \((2, \infty)\)
(c) \((-\infty, 2)\)
(d) \((-\frac{1}{3}, \infty)\)
(e) \((-\infty, \frac{1}{3})\)

\[ f'(x) = 12x^2 - 20x - 8 \]
\[ = 4(3x^2 - 5x - 2) \]
\[ = 4(3x + 1)(x - 2) \]
\[ f'(x) = 0 \implies x = -\frac{1}{3} \text{ or } x = 2 \]

2. The function \( f(x) = x^4 - 8x^2 \) has a relative maximum when \( x = \)

(a) 0
(b) -1
(c) -2
(d) 1
(e) 2

\[ f'(x) = 4x^3 - 16x \]
\[ = 4x(x^2 - 4) \]
\[ = 4x(x-2)(x+2) \].

\[ f'(x) = 0 \implies x = 0 \text{ or } x = 2 \text{ or } x = -2. \]
3. On the interval \([-1, 1]\), the function \(f(x) = 4 + x^2 - x^3\) has an absolute maximum when \(x = \) 

(a) \(-1\)  
(b) 1  
(c) 0  
(d) \(\frac{1}{2}\)  
(e) \(-\frac{1}{2}\)

\[f'(x) = 2x - 3x^2\]  
\[f'(x) = 0 \Rightarrow x = 0 \text{ or } x = \frac{2}{3}\]

\[f\left(\frac{2}{3}\right) = 4 + 4 \left(\frac{2}{3}\right) = \frac{8}{3}\]

\[f(0) = 4\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
<th>(\text{abs. max})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>(-\frac{1}{2})</td>
<td>(\frac{11}{4})</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td></td>
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<tr>
<td>-1</td>
<td>6</td>
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4. The number of inflection points for the function \(f(x) = \frac{x^5}{20} + \frac{x^4}{12} + x - 3\) is 

(a) 1  
(b) 2  
(c) 3  
(d) 4  
(e) 0

\[f''(x) = x^3 + x^2\]
\[f''(x) = 0 \Rightarrow x = 0 \text{ or } x = -1\]

\(f''(x)\) is positive for \(x > 0\) and negative for \(x < 0\).

\((-1, f(-1))\) is the only inflection point.
5. The function \( f(x) = \frac{x^2 - 1}{x} \) is concave up on

- (a) \((-\infty, 0)\)
- (b) \((-\infty, 1)\)
- (c) \((1, \infty)\)
- (d) \((0, \infty)\)
- (e) \((-\infty, 0)\) and \((0, 2)\)

\[ f(x) = x - \frac{1}{x} \]
\[ f'(x) = 1 + \frac{1}{x^2} = 1 + x^{-2} \]
\[ f''(x) = -2x^{-3} = -\frac{2}{x^3} \]

6. The revenue equation for a company is given by \( r(q) = 1296q - 0.12q^3 \). A relative extrema for \( r(q) \) on the interval \((0, \infty)\) occurs when the number of units \( q = \)

- (a) 60
- (b) 70
- (c) 80
- (d) 90
- (e) 100

\[ r'(q) = 1296 - 0.36q^2 \]
\[ r'(q) = 0 \implies q^2 = \frac{1296}{36} = 36 \]
\[ q = 60 \] (Notice \( q > 0 \)).
7. An equation of a horizontal asymptote for the graph of \( y = \frac{2x}{9x^2 - 1} \) is

(a) \( y = 0 \)

\[
\lim_{x \to \pm\infty} \frac{2x}{9x^2 - 1} = \lim_{x \to \pm\infty} \frac{2}{9x} = 0
\]

(b) \( y = \frac{1}{3} \)

(c) \( x = \frac{2}{9} \)

(d) \( x = \frac{1}{3} \)

(e) \( y = \frac{2}{9} \)

\[ y = 0 \] is the only horizontal asymptote.

8. The number of vertical asymptotes for the graph of \( y = \frac{2x}{4x^2 - x} \) is

(a) 1

\( 4x^2 - x = 0 \rightarrow x(4x - 1) = 0 \)

\( x = 0 \) or \( x = \frac{1}{4} \).

(b) 2

(c) 3

(d) 0

(e) \( \infty \)

\[ x = \frac{1}{4} \] is the only V.A.
9. The demand equation for a monopolist’s product is \( p = \frac{500}{\sqrt{q}} \), where \( p \) is the price per unit (in dollars) for \( q \) units. If the total cost \( c \) (in dollars) of producing \( q \) units is given by \( c = 5q + 2000 \), then the level of production that maximizes the profit is

(a) 2500 units
(b) 1235 units
(c) 750 units
(d) 100 units
(e) 50 units

The revenue is
\[ r(q) = pq = 2 \cdot \frac{500}{\sqrt{q}} = 500\sqrt{q} \]

The profit is
\[ \pi(q) = r(q) - c = 500\sqrt{q} - 5q - 2000 \]

\[ \pi'(q) = 0 \quad \Rightarrow \quad q = 2500 \]

10. A rectangular plot adjacent to a stream is to be fenced in by using the stream as one side of the enclosed area. If 2000 ft of fencing are to be used, find the maximum area that can be enclosed. Assume the answer is in square feet.

(a) 500,000
(b) 700,000
(c) 800,000
(d) 600,000
(e) 400,000

\[ x + 2y = 2000 \quad \Rightarrow \quad y = \frac{2000 - x}{2} \]

\[ A = xy = x \left( \frac{2000 - x}{2} \right) = 1000x - \frac{x^2}{2} \]

\[ A'(x) = 1000 - x \quad \Rightarrow \quad A''(x) = -1 \]

At \( x = 1000 \), the area is
\[ A = 500,000 \]
11. The estimate change in \( f(x) = x^3 - 3x^2 \) when \( x \) changes from 1 to 1.1 is

\[(a) \quad -0.3 \]

\[f'(x) = 3x^2 - 6x \]

\(dy \approx dy = f'(x) \, dx \)

\[(b) \quad 0.3 \]

\[(c) \quad -0.6 \]

\[(d) \quad 0.6 \]

\[(e) \quad 0 \]

\(\frac{f'(1)}{1.1 - 1} = (-3)(1.1 - 1) = -0.3\).

12. If \( q = \frac{1}{p^3} \), then \( \frac{dp}{dq} = \)

\[(a) \quad -\frac{p^4}{3} \]

\[(b) \quad -\frac{3}{p^4} \]

\[(c) \quad \frac{p^3}{2} \]

\[(d) \quad \frac{2}{p^3} \]

\[(e) \quad \frac{1}{p} \]

\[\frac{d}{dp} = -\frac{3}{p^4} \]

\[\frac{d}{dx} = \frac{1}{dx/\,dp} = -\frac{3}{p^4} \]

\[= -\frac{3}{p^4} \]
13. \[ \int 5e^{-x} \, dx = \]

- \( a) -5e^{-x} + C \)
- \( b) -5e^x + C \)
- \( c) \frac{1}{5}e^{-x} + C \)
- \( d) 5e^x + C \)
- \( e) 5e^{-x} + C \)

14. \[ \int \frac{d}{dx}(\sqrt{x^3 + 2}) \, dx = \]

- \( a) \sqrt{x^3 + 2} + C \)
- \( b) 3x^2 \sqrt{x^3 + 2} + C \)
- \( c) \frac{3x^2}{\sqrt{x^3 + 2}} + C \)
- \( d) \frac{\sqrt{x^3 + 2}}{3x^2} + C \)
- \( e) \sqrt{3x^2} + 2 + C \)
15. Let the marginal cost function be

$$\frac{dc}{dq} = 0.003 q^2 - 0.6q + 40$$

where $q$ is the number of units produced. If the fixed costs are 8000, the average cost when producing 100 units is

(a) 100

$$C(q) = \int (0.003 q^2 - 0.6q + 40) \, dq$$

(b) 90

$$= 0.003 \frac{q^3}{3} - 0.6 \frac{q^2}{2} + 40q + d$$

(c) 80

$$= 0.001 q^2 - 0.3 q^2 + 40q + 8000$$

(d) 70

$$\text{(Since the fixed cost is } C(0))$$

$$\bar{c} = \frac{c}{q} = 0.001 q - 0.3 q^2 + 40 + \frac{8000}{q}$$

(e) 60

$$\frac{C'(100)}{100} = \frac{1}{100} (100)^2 - \frac{3}{10} (100) + 40 + \frac{8000}{100} = 10 - 30 + 40 + 80 = 100$$

16. Let

$$y'' = 20x^3 + e^x, \quad y'(0) = 2 \quad \text{and} \quad y(0) = -1.$$ 

Then $y(1) = \ldots$

(a) $e$

$$y' = \int y^2 \, dx = \int (20x^3 + e^x) \, dx$$

(b) $2e$

$$= 20 \frac{x^4}{4} + e^x + c.$$ 

(c) $-e$

$$y'(0) = 2 \quad \Rightarrow \quad 1 + c = 2 \quad \Rightarrow \quad c = 1.$$ 

(d) $e + 1$

$$y = 5x^4 + e^x + 1$$

(e) $e - 1$

$$\Rightarrow \quad y(0) = -1 \quad \Rightarrow \quad 1 + d = -1 \quad \Rightarrow \quad d = -2.$$
17. Let \( y'(x) = 3\sqrt{x+3} \) and \( y(1) = 0 \). Then \( y(6) = \)

(a) 38

\[
y = \int y' \, dx = \int 3(x+3)^{\frac{1}{2}} \, dx
\]

\[
= 3 \cdot \frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}} + C
\]

\[
= 2 \sqrt{(x+3)^3} + C.
\]

\( y(1) = 0 \) \( \Rightarrow \) \( 2(8) + C = 0 \)

\( \Rightarrow C = -16. \)

\[
y = 2 \sqrt{(x+3)^3} - 16.
\]

(b) 32

(c) 28

(d) 14

(e) 9

18. \( \int \frac{4x + 2}{x^2 + x + 3} \, dx = \)

(a) \( 2 \ln (x^2 + x + 3) + C \)

(b) \( \ln (x^2 + x + 3) + C \)

(c) \( \frac{1}{2} \ln (x^2 + x + 3) + C \)

(d) \( 2 \ln (x+1) + C \)

(e) \( \frac{1}{2} \ln (x+1) + C \)

\[
I = \int \frac{2(2x+1)}{x^2 + x + 3} \, dx
\]

\[
= 2 \int \frac{du}{u}
\]

\[
= 2 \ln |u| + C.
\]

Since the determinant

\[
b^2 - 4ac = 1 - 12 < 0
\]

\( \Rightarrow \) No need for \( x \).
19. A certain country’s marginal propensity to save is given by

\[
\frac{dS}{dI} = \frac{10}{(I + 3)^2},
\]

where \( S \) and \( I \) represent the total national savings and income, respectively, and are measured in billions of dollars. If total national consumption is 6 million when the total national income is 7 billion, then the value of \( I \) that makes the total national savings equal to zero is

(a) 2
(b) 3
(c) 1
(d) 4
(e) 7

20. If \( y' = \frac{x}{x + 1} \) and \( y(0) = 1 \), then \( y(1) = \)

(a) \( 2 - \ln 2 \)
(b) \( 2 + \ln 2 \)
(c) \( 2 \)
(d) \( \ln 2 \)
(e) \( 1 + \ln 2 \)