King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 201

Exam II

182

Wednesday 20/3/2019

Net Time Allowed: 120 minutes

MASTER VERSION

1. The domain of the function $f(x,y) = \arcsin(x^2+y^2-3)$ is
(**Note: the circles are given in polar coordinates**)

(a) all points (x,y) on and between the circles $r=2$ and $r=\sqrt{2}$

(b) all points (x,y) outside the circle $r=2$

(c) all the points (x,y) inside $r=2$

(d) all the points (x,y) inside $r=\sqrt{2}$

(e) all the points (x,y) outside $r=\sqrt{2}$

2. $\lim\limits_{(x,y)\to(-1,1)} \dfrac{x^2y-y^3}{x+y}$ equals

(a) -2

(b) 0

(c) 1

(d) 2

(e) does not exist

3. The equation of the tangent plane of $2z^2 = x^2 + y^2$ at $(1, 1, 1)$ is:

(a) $x + y - 2z = 0$

(b) $2x + 2y - z = 3$

(c) $x + y + 2z = 4$

(d) $2x + 2y + z = 5$

(e) $3x - y + z = 3$

4. Let $w = xy + yz + zx$, $x = r\cos\theta$, $y = r\sin\theta$, $z = r\theta$.
Then $\dfrac{\partial w}{\partial \theta}$ when $r = 2$ and $\theta = \dfrac{\pi}{2}$ is

(a) -2π

(b) 2π

(c) 0

(d) π

(e) $-\pi$

5. The directional derivative of $f(x,y) = \dfrac{x^2}{x+y}$ at the point $(1,1)$ in the direction of the vector $\vec{v} =< -3, 4 >$ is

(a) $-\dfrac{13}{20}$

(b) $\dfrac{7}{20}$

(c) $-\dfrac{3}{20}$

(d) $-\dfrac{4}{5}$

(e) $\dfrac{3}{5}$

6. Which one of the following statement is **wrong** about the surface $y^2 + z^2 - 4y + 2z - x + 1 = 0$?

(a) its trace on $xz-$ plane is an ellipse

(b) it is an elliptic paraboloid

(c) it has a vertex at $(-4, 2, -1)$

(d) its trace on$yz-$ plane is a circle

(e) its trace on$xy-$ plane is a parabola

7. Consider the following statements about the surface $x = \sqrt{4y^2 + z^2 - 8y + 2z + 9}$
(I) Its graph has two sheets.
(II) It has a vertex at $(0, 1, -1)$
(III) Its axis is parallel to x-axis.
Which of the above statements is (are) **true**?

(a) only III

(b) II and III

(c) I, II and III

(d) I and II

(e) only II.

8. The distance from the plane $2z = x + 2y - 1$ to the plane $z = \frac{x}{2} + y - 4$ is

(a) $\frac{7}{3}$

(b) 3

(c) 2

(d) 0

(e) $\frac{8}{3}$

9. An equation of the plane through the point $p(1, -2, 3)$ that contains the line $\dfrac{x}{2} = y - 1,\ z = -1$ is

(a) $4x - 8y - 7z = -1$

(b) $2x + 4y - 3z = 1$

(c) $-4x - 8y + 7z = 33$

(d) $2x + y = 0$

(e) $2x + y + z = 3$

10. $\displaystyle\lim_{(x,y)\to(1,1)} \frac{\sqrt{1+2x} - \sqrt{1+2y}}{xy + 2x - y^2 - 2y} =$

(a) $\dfrac{\sqrt{3}}{9}$

(b) $4\sqrt{3}$

(c) $\dfrac{2\sqrt{3}}{3}$

(d) 0

(e) does not exist

11. The set of **all** points in R^2 at which $f(x) = \dfrac{x^2 + y^2}{1 + \ln(e^{-1} + x - y)}$ is continuous is

(a) $\{(x, y) \in R^2 : y \neq x \text{ and } y < x + e^{-1}\}$

(b) $\{(x, y) \in R^2 : e^{-1} + x - y > 0\}$

(c) $\{(x, y) \in R^2 : y < x\}$

(d) $\{(x, y) \in R^2 : x < y < x + e^{-1}\}$

(e) $\{(x, y) \in R^2 : x \in (-\infty, \infty),\ y \in (-\infty, \infty)\}$

12. Consider the function

$$f(x, y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Which of the following statements is **TRUE**?

(a) f is discontinuous at $(0, 0)$

(b) f is discontinuous at $(1, 0)$

(c) $f(x, y) \to 2$ as $(x, y) \to (0, 0)$ along the curve $y = x^2$

(d) $\lim\limits_{(x,y)\to(0,0)} f(x, y) = 0$

(e) $f(x, y) \to 1$ as $(x, y) \to (0, 0)$ along the line $y = x$

13. If $f(x,y) = [\tan^{-1}(xy)]^4$, then the value of $f_x(1,1)$ is

(a) $\dfrac{\pi^3}{32}$

(b) $\dfrac{\pi^3}{64}$

(c) $\dfrac{\pi^3}{16}$

(d) $\dfrac{\pi^3}{4}$

(e) π^3

14. If $F(x,y) = \int_x^y e^{\cos t}\,dt$, then $F_y - F_x =$

(a) $e^{\cos y} + e^{\cos x}$

(b) $2\,e^{\cos x}$

(c) $2\,e^{\cos y}$

(d) $e^{\cos x} - e^{\cos y}$

(e) 0

15. If $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$, then $u_{xx} + u_{yy} + u_{zz} =$

(a) 0

(b) $\dfrac{3}{(x^2+y^2+z^2)^{3/2}}$

(c) $\dfrac{3+3x}{(x^2+y^2+z^2)^{5/2}}$

(d) $\dfrac{3-3x}{(x^2+y^2+z^2)^{3/2}}$

(e) $\dfrac{3x}{(x^2+y^2+z^2)^{5/2}}$

16. Let $yz + x \ln y = z^2$. Then $\dfrac{\partial z/\partial x}{\partial z/\partial y}$ equals

(where z is given implicitly as a function of x and y).

(a) $\dfrac{y \ln y}{yz+x}$

(b) $\dfrac{\ln y}{yz+x}$

(c) $\dfrac{y \ln y}{yx+z}$

(d) $\dfrac{y}{yz+x}$

(e) $\dfrac{x+zy}{y \ln y}$

17. Let $f(x,y) = x^2\, e^{xy}$. The direction in which f has the maximum rate of change at $(1,0)$ is

(a) $\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$

(b) $\left\langle \frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle$

(c) $\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

(d) $\left\langle \frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \right\rangle$

(e) $\left\langle \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right\rangle$

18. The base radius and height of a right circular cone are measured as $5\,cm$ and $15\,cm$ respectively, with a possible error in measurement of as much as $0.1\,cm$ in each. Use differentials to estimate the maximum possible error in the calculated volume of the cone.

(a) $\frac{35\,\pi}{6}$

(b) $20\,\pi$

(c) $15\,\pi$

(d) $\frac{10\,\pi}{3}$

(e) $\frac{5\,\pi}{6}$

19. If $z = y + f\left(x^2 - y^2\right)$, where f is differentiable, then $y\dfrac{\partial z}{\partial x} + x\dfrac{\partial z}{\partial y} =$

(a) x

(b) y

(c) z

(d) $x + y$

(e) 0

20. If the normal line to the surface $z = x^2 - y^2 + 1$ at the point $(2, 0, 5)$ intersects the surface at a second point (a, b, c), then $16c + b - 4a =$

(a) 106

(b) 0

(c) -40

(d) 32

(e) -201

Q	MM	V1	V2	V3	V4
1	a	a	e	c	b
2	a	b	e	d	d
3	a	c	b	a	b
4	a	b	e	e	b
5	a	c	c	e	a
6	a	d	a	e	c
7	a	d	c	a	d
8	a	a	b	b	a
9	a	c	d	d	a
10	a	b	d	e	a
11	a	c	b	e	e
12	a	d	b	a	d
13	a	d	c	d	e
14	a	e	a	a	a
15	a	a	e	c	a
16	a	e	c	d	e
17	a	d	a	d	a
18	a	c	b	c	e
19	a	a	d	d	a
20	a	d	c	d	e