1. The range of the function $f(x, y) = \sqrt{9 - x^2 - y^2}$ is

(a) $[0, 3]$

(b) $(0, 3) \cup (3, 9)$

(c) $(0, \infty)$

(d) $(-\infty, 0)$

(e) $(-3, 3)$

2. A polar coordinates $(r, \theta)$ of the point $(\sqrt{3}, 1)$ with $r < 0$ are

(a) $\left(-2, \frac{7\pi}{6}\right)$

(b) $\left(-2, \frac{4\pi}{3}\right)$

(c) $\left(-2, \frac{5\pi}{6}\right)$

(d) $\left(-2, \frac{\pi}{6}\right)$

(e) $\left(-2, \frac{5\pi}{4}\right)$

3. Given the curve

$$x = 2\sin t, \quad y = 3\cos t, \quad 0 \leq t \leq \pi$$

the curve is concave upward on the interval

(a) $\left(\frac{\pi}{2}, \pi\right)$

(b) $\left(0, \frac{\pi}{2}\right)$

(c) $(0, \pi)$

(d) $\left(0, \frac{\pi}{4}\right)$

(e) $\left(\frac{\pi}{4}, \pi\right)$

4. The parametric curve $x = \sin t$, $y = \sin^2 t$, $\frac{-\pi}{2} \leq t \leq 3\pi$ passes by the origin

(a) 4 times

(b) 5 times

(c) 6 times

(d) 3 times

(e) 2 times

5. The polar equation

$$r = a\sin\theta + b\cos\theta$$

where $a, b$ not both zero, represents

(a) a circle with center $\left(\frac{b}{2}, \frac{a}{2}\right)$

(b) a circle with center $(a, b)$

(c) a circle with center $\left(\frac{a}{2}, \frac{b}{2}\right)$

(d) a circle with center $\left(-\frac{b}{2}, -\frac{a}{2}\right)$

(e) a circle with center $\left(-\frac{a}{2}, -\frac{b}{2}\right)$

6. The area of the region outside the circle $r = 1$ and inside the circle $r = 2\cos\theta$ is

(a) $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$

(b) $\pi - \sqrt{3}$

(c) $\frac{\pi}{2} + \sqrt{3}$

(d) $\frac{\pi}{4} - \frac{\sqrt{3}}{3}$

(e) $\frac{\pi}{6} + \frac{\sqrt{3}}{2}$

7. The angle between a unit vector $\vec{a}$ and a vector $\vec{b}$ is $\frac{\pi}{3}$. If $|\vec{b}| = 2$, then the length of $\vec{a} + \vec{b}$ equals

(a) $\sqrt{7}$

(b) 3

(c) $\sqrt{3}$

(d) 7

(e) 5

8. If $\vec{v}$ is the vector projection of $\vec{a} =< -1, 1, 1 >$ onto $\vec{b} =< 2, 0, -3 >$, then $\vec{a} - 13\vec{v} =$

(a) $< 9, 1, -14 >$

(b) $< 2, 0, -3 >$

(c) $< -11, 1, 16 >$

(d) $< 10, 0, -15 >$

(e) $< -3, 1, 4 >$

9. If the line $x = 2+3t,\ y = -4t,\ z = 5+t$ intersects the plane $2x-y+z = -2$ at $(a, b, c)$ then $a + b + c =$

(a) 7

(b) 8

(c) 5

(d) $-2$

(e) 4

10. The graph of the equation

$$-2x^2 + 8x + y + 5z^2 = 8$$

is

(a) a hyperbolic paraboloid

(b) an elliptic paraboloid

(c) an elliptic cone

(d) a hyperboloid of one sheet

(e) a hyperboloid of two sheets

11. $\displaystyle\lim_{L=(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} =$

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) 2

(e) does not exist

12. The volume of the parallelepiped determined by the vectors

$$\vec{a} =< 1, 1, -2 >, \vec{b} =< 3, -5, 1 >, \vec{c} =< -4, 1, 1 >$$

is equal to

(a) 21

(b) 13

(c) 34

(d) 50

(e) 46

13. If $H(x, y, z) = x^2 e^{y/z} + \sin(yz)$, then the value of $H_x + H_y + H_z$ at the point $(1, 0, 1)$ is

(a) 4

(b) 2

(c) 3

(d) 1

(e) 0

14. If $u = xz^2 + y \ln(xy + z)$, $x = s^2 + t$, $y = e^{st}$, $z = s \tan^{-1} t$, then the value of $\frac{\partial u}{\partial t}|_{(s,t)=(1,0)}$ is

(a) 3

(b) 2

(c) 4

(d) 1

(e) 0

15. Which one of the following statement(s) is (are) **True**?
(I). The set of points $\{(x, y, z)|x^2 + y^2 = 1\}$ is a circle.
(II). The set of all points whose distances from the point $(0, 4, 0)$ are equal to their distances from the origin is a sphere.
(III). $x^2 + y^2 + z^2 - 8x + 2y + 6z + 1 = 0$ represents a sphere with radius 5.

(a) III only

(b) II and III

(c) I, II and III

(d) I and II

(e) II only

16. Evaluate $\int\int_R y \sin xy \, dA$, where $R = [1, 2] \times \left[0, \frac{\pi}{2}\right]$ is equal

(a) 1

(b) $\pi$

(c) 0

(d) $-1$

(e) $-\pi$

17. The function $f(x,y) = 3xy - x^2y - xy^2$ has

(a) three saddle points

(b) two saddle points

(c) one saddle point

(d) no saddle point

(e) four saddle points

18. The differential $dz$ of the function $z = f(x,y) = x \ln y$ as $(x,y)$ moves from $(1,1)$ to $(0.9, 1.1)$ is equal to

(a) 0.1

(b) $-0.1$

(c) $-1$

(d) 1

(e) 0

19. The point of intersection between the normal line to the surface $x^3yz = 4x$ at the point $(1, 2, 2)$ and the plane $x + y - z = 9$ is

(a) $(9, 4, 4)$

(b) $(9, 2, 2)$

(c) $(8, 3, 2)$

(d) $(8, -2, 3)$

(e) $(2, -2, -9)$

20. The maximum value of $f(x, y) = xy$ on the ellipse $x^2 + 4y^2 = 4$ is

(a) 1

(b) 0

(c) $-1$

(d) 2

(e) 3

21. Evaluate the integral by reversing the order of integration

$$\int_0^9 \int_{\sqrt{y}}^3 e^{x^3}\, dx\, dy =$$

(a) $\dfrac{e^{27}-1}{3}$

(b) $\dfrac{e^{27}}{3}$

(c) $\dfrac{e^9-1}{9}$

(d) $\dfrac{e^9-1}{3}$

(e) $\dfrac{e^9}{9}$

22. By converting to polar coordinates,

$$\int_{-5}^5 \int_0^{\sqrt{25-x^2}} (x^2+y^2)^{3/2}\, dy\, dx =$$

(a) $5^4\,\pi$

(b) $2(5)^4\,\pi$

(c) $5^5\,\pi$

(d) $\dfrac{\pi}{5}$

(e) $\dfrac{4^5\pi}{5}$

23. If the volume of the tetrahedron bounded by the planes:

$$x + y + z = 2,\ x = y,\ x = 0 \text{ and } z = 0$$

is given by $\int_0^d \int_{ax}^{2+bx} \int_0^{2-x-y} c\, dz\, dy\, dx$ then $a + b + c + d =$

(a) 2

(b) 1

(c) $-1$

(d) 3

(e) 0

24. The volume of the solid enclosed by the paraboloid $z = 8 - 2x^2 - 2y^2$ and the plane $z = 2$ is

(a) $9\pi$

(b) $\frac{7}{2}\pi$

(c) $\frac{7}{3}\pi$

(d) $5\pi$

(e) $\frac{9}{2}\pi$

25. The volume of the region that lies between the paraboloid $z = 24 - x^2 - y^2$ and the cone $z = 2\sqrt{x^2 + y^2}$ is given by:

(a) $\int_0^{2\pi} \int_0^{4} \int_{2r}^{24-r^2} r \, dz \, dr \, d\theta$

(b) $\int_0^{2\pi} \int_0^{\cos\theta} \int_{2r}^{24-r^2} r \, dz \, dr \, d\theta$

(c) $\int_0^{2\pi} \int_0^{6} \int_{2r}^{24-r^2} r \, dz \, dr \, d\theta$

(d) $\int_0^{2\pi} \int_0^{4} \int_{24-r^2}^{2r} r \, dz \, dr \, d\theta$

(e) $\int_0^{2\pi} \int_0^{4} \int_{4r^2}^{\sqrt{24-r^2}} r \, dz \, dr \, d\theta$

26. Evaluate $\int\int_E\int z \, dV$, where $E$ lies inside the sphere $\rho = 1$ and above the cone $\phi = \dfrac{\pi}{4}$ in the first octant

(a) $\dfrac{\pi}{32}$

(b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{8}$

(d) $\dfrac{\pi}{2}$

(e) $\dfrac{\pi}{16}$

27. The integral $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dx\, dy$ can be written as $\int_0^a \int_0^b \int_0^c \rho^k \sin^m \varphi \, d\rho \, d\theta \, d\varphi$, then the value of $\left[\frac{ac}{bk}\right]^m$

(a) $\frac{1}{3}$

(b) 2

(c) 4

(d) 1

(e) $\frac{1}{2}$

28. The absolute min of

$$f(x,y) = x^2 - 4x + y^2 - 2y$$

on the region bounded by $x + y = 4$ and $y = x + 4$ and the $x$-axis is

(a) $-5$

(b) $-16$

(c) $-4$

(d) $\frac{-9}{2}$

(e) 0

| Q | MM | V1 | V2 | V3 | V4 |
|---|---|---|---|---|---|
| 1 | a | d | d | b | a |
| 2 | a | a | e | e | a |
| 3 | a | d | b | c | d |
| 4 | a | a | b | e | e |
| 5 | a | a | b | a | e |
| 6 | a | b | d | a | c |
| 7 | a | e | e | a | d |
| 8 | a | d | d | e | b |
| 9 | a | a | b | e | d |
| 10 | a | c | e | b | d |
| 11 | a | b | b | b | a |
| 12 | a | d | b | c | b |
| 13 | a | c | a | e | b |
| 14 | a | d | b | a | e |
| 15 | a | d | d | b | d |
| 16 | a | e | c | e | b |
| 17 | a | e | c | e | b |
| 18 | a | e | a | a | d |
| 19 | a | c | a | a | c |
| 20 | a | d | c | b | b |
| 21 | a | b | c | e | b |
| 22 | a | b | d | e | a |
| 23 | a | e | e | b | d |
| 24 | a | a | c | d | d |
| 25 | a | c | c | a | c |
| 26 | a | a | d | a | c |
| 27 | a | a | b | e | e |
| 28 | a | b | a | b | c |