Math201.3, Quiz # 2, Term 182

Name:

Solutions

ID#: **Serial #:**

1. **[2.5 points]** Find the point of intersection, if it exists, between the following two lines:

$$L_1: x = 1 + t, \quad y = -2 + 3t, \quad z = 2 + 5t$$
$$L_2: x = 2s, \quad y = 3 + s, \quad z = 5 + 5s\,.$$

2. **[3 points]** Identify (name, vertex, axis) and sketch the surface

$$4x^2 - y^2 + 2y + 2z^2 + 3 = 0.$$

3. **[2.5 points]** Let $f(x,y) = \frac{3}{x-y^2}$.
   a. Find and sketch the domain of f.
   b. Find an equation for the level curve of f that passes through the point $(-2,1)$. Sketch the level curve.

4. **[2 points]** Find the limit if it exists: $\lim\limits_{(x,y)\to(1,1)} \frac{x^3-y}{x-y}$.

Good luck,

Ibrahim Al-Rasasi

0.5

$\boxed{1}$ Equate

$x=x$ $\quad 1+t = 2s$ — (1)

$y=y \Rightarrow -2+3t = 3+s$ — (2)

$z=z$ $\quad 2+5t = 5+5s$ — (3)

$(1) \Rightarrow s = \frac{1}{2} + \frac{1}{2}t$ (*)

$\overset{(2)}{\Longrightarrow} -2+3t = 3+\frac{1}{2}+\frac{1}{2}t$

$\Rightarrow -4+6t = 6+1+t$

$\Rightarrow 5t = 11$ 0.5

$\Rightarrow t = \frac{11}{5}$

$\overset{*}{\Rightarrow} s = \frac{1}{2}+\frac{1}{2}\cdot\frac{11}{5} = \frac{16}{10} = \frac{8}{5}$ 0.5

Check (3):

$2+5\cdot\frac{11}{5} = 5+5\cdot\frac{8}{5}$

$2+11 = 5+8$

$13 = 13$ ✓ ok 0.5

The two Lines Intersect:

$t=\frac{11}{5} \overset{L_1}{\Longrightarrow} x = 1+\frac{11}{5} = \frac{16}{5}$

$y = -2+3\cdot\frac{11}{5} = \frac{23}{5}$

$z = 2+5\cdot\frac{11}{5} = 13$

the point of intersection is

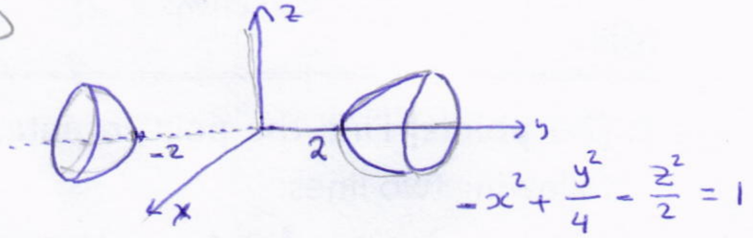
$\left(\frac{16}{5}, \frac{23}{5}, 13\right)$. 0.5

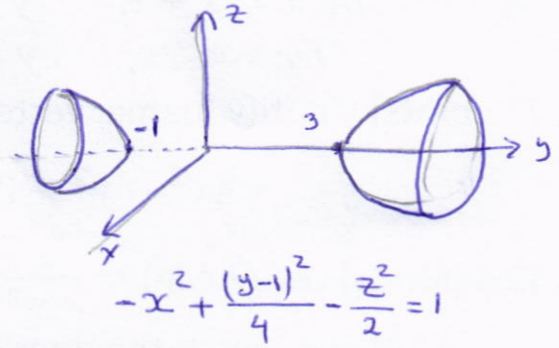
2 $4x^2 - y^2 + 2y + 2z^2 + 3 = 0 \Rightarrow 4x^2 - (y^2 - 2y + 1) + 2z^2 = -3 - 1$

$\Rightarrow 4x^2 - (y-1)^2 + 2z^2 = -4 \Rightarrow -x^2 + \frac{(y-1)^2}{4} - \frac{z^2}{2} = 1$ 1

- a hyperboloid of two sheets 0.5
- vertecs: $x = 0, z = 0 \Rightarrow \frac{(y-1)^2}{4} = 1 \Rightarrow (y-1)^2 = 4 \Rightarrow y - 1 = \pm 2 \Rightarrow y = -1, 3$

  $\Rightarrow (0, -1, 0), (0, 3, 0)$ 0.5
- axis: the y-axis. 0.5



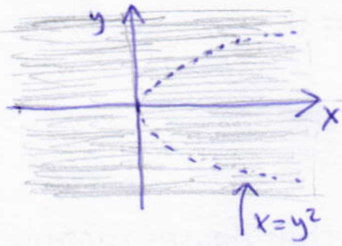


0.5

3 $f(x,y) = \frac{3}{x - y^2}$

(a) Domain $= \{(x,y) \in \mathbb{R}^2 : x - y^2 \neq 0\}$

$= \{(x,y) \in \mathbb{R}^2 : x \neq y^2\}$ 0.5

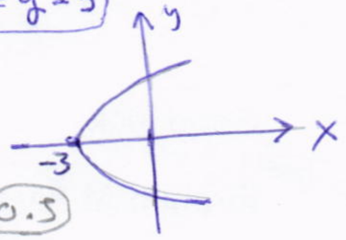


0.5

(b) Eq. of the level Curve is $f(x,y) = c$, or, $\frac{3}{x - y^2} = c$. 0.5

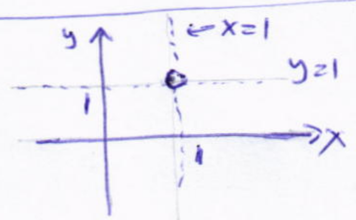
Find the value of c? Subsitite $(-2, 1)$: $\frac{3}{-2 - (1)^2} = c \Rightarrow c = -1$ 0.5

The equation is $\frac{3}{x - y^2} = -1$, or, $-3 = x - y^2$, or $\boxed{x = y^2 - 3}$



0.5

4 $\lim_{(x,y) \to (1,1)} \frac{x^3 - y}{x - y}$



- along the line $x = 1$:

$\lim_{\substack{(x,y) \to (1,1) \\ x = 1}} \frac{x^3 - y}{x - y} = \lim_{(x,y) \to (1,1)} \frac{x^3 - y}{x - y}\Big|_{x=1} = \lim_{y \to 1} \frac{1 - y}{1 - y} = \lim_{y \to 1} 1 = 1$ 0.5

1

- along the line $y = 1$:

$\lim_{\substack{(x,y) \to (1,1) \\ y = 1}} \frac{x^3 - y}{x - y} = \lim_{(x,y) \to (1,1)} \frac{x^3 - y}{x - y}\Big|_{y=1} = \lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} (x^2 + x + 1) = 1 + 1 + 1 = 3$

- Since the limits are <u>not equal</u> along the two paths, then

$\lim_{(x,y) \to (1,1)} \frac{x^3 - y}{x - y}$ DNE. 0.5

Math201.07, Quiz #2, Term 182

Name: Solutions

ID#: **Serial #:**

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1. **[2.5 points]** Find an equation of the plane that contains the line
$$L: x = 1 + t, \qquad y = 2 - t, \qquad z = 4 - 3t$$
and parallel to the plane $5x - 2y + z = 3$.

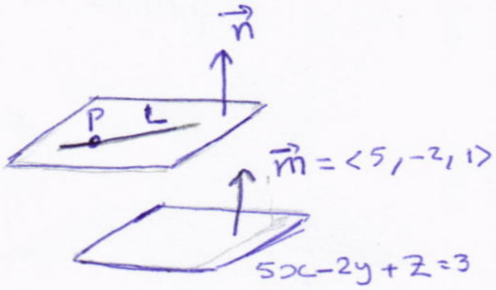
2. **[3 points]** Identify (name, vertex, axis) and sketch the surface
$$4x^2 - 3y^2 + 6y + 2z^2 - 3 = 0.$$

3. **[2.5 points]** Find and sketch the domain of $f(x, y) = \frac{\ln(2+x)}{2-x^2-y^2}$.

4. **[2 points]** Find the limit if it exists: $\lim\limits_{(x,y)\to(0,0)} \frac{x+\sin(2y^2)}{2x+y^2}$.

Good luck,

Ibrahim Al-Rasasi



$\boxed{1}$ • a point on the plane : We can take any point on the line L. If $t=0$, we get the point $P(1, 2, 4)$ (0.5)

• Since the two planes are parallel, then their normals are parallel. So

(1) a normal vector to the plane = a normal vector to the plane $5x - 2y + z = 3$

$$\vec{n} = \vec{m} = \langle 5, -2, 1 \rangle$$

• An equation for the plane is

$$5(x-1) - 2(y-2) + 1(z-4) = 0 \quad (0.5)$$

$$\Rightarrow \quad 5x - 5 - 2y + 4 + z - 4 = 0$$

$$\Rightarrow \quad 5x - 2y + z = 5 \quad (0.5)$$