MATH 202.14 (Term 182)

Quiz 1 (Sects. 2.2 & 2.3) Duration: 20min

Name: ID number:

1.)(5pts)Solve the DE: $y\frac{dy}{dx} = (y-2)(y+1)\cos^3 x$.

2.)(5pts)Solve the IVP: $\begin{cases} \frac{dy}{dx} + \frac{1}{2\sqrt{x-1}}y = \frac{1}{2\sqrt{x-1}}, \\ y(2) = 0. \end{cases}$

1) $y = c$

$\Rightarrow 0 = (c-2)(c+1)\cos^3 x \,,\ \forall x \in I$

$\Rightarrow c = 2 \,,\ c = -1$

$y = 2$ and $y = -1$ are constant solutions

Now, assume $y \neq 2, -1$

The equation is seperable

$$\underbrace{\int \frac{y}{(y-2)(y+1)}}_{\text{partial fraction}} dy = \int \cos^3 x \, dx$$

$$= \frac{a}{y-2} + \frac{b}{y+1} \Rightarrow \begin{matrix} a = 2/3 \\ b = 1/3 \end{matrix}$$

Thus,

$$\int \left(\frac{2/3}{y-2} + \frac{1/3}{y+1} \right) dy = \underbrace{\int \cos x (1 - \sin^2 x) dx}_{\text{substitution } u = \sin x}$$

$$\frac{2}{3} \ln|y-2| + \frac{1}{3} \ln|y+1| = \sin x - \frac{\sin^3 x}{3} + C$$

$$\ln|(y-2)^2(y+1)| = 3\sin x - \sin^3 x + C$$

$$(y-2)^2(y+1) = c\, e^{3\sin x - \sin^3 x}$$

implicit solution

2.) This is a Linear DE

$$e^{\int \frac{dx}{2\sqrt{x-1}}} = e^{\sqrt{x-1}}$$

We multiply both sides by $e^{\sqrt{x-1}}$!

$$\Rightarrow \frac{d}{dx}\left(y e^{\sqrt{x-1}} \right) = \frac{1}{2\sqrt{x-1}} e^{\sqrt{x-1}}$$

We integrate with respect to x.

$$y e^{\sqrt{x-1}} = \underbrace{\int \frac{1}{2\sqrt{x-1}} e^{\sqrt{x-1}} dx}_{\text{substitution } u = \sqrt{x-1}}$$

$$= e^{\sqrt{x-1}} + C$$

$$\Rightarrow y = 1 + c e^{-\sqrt{x-1}}$$

$y(2) = 0$

$\Rightarrow 0 = 1 + c e^{-1} \,,\ c = -e$

$$\Rightarrow y = 1 - e^{1 - \sqrt{x-1}} \,,\ x \in (1, \infty)$$