1.) (5 pts) Solve the DE: \( y' + (y - 2)(y + 1) \cos^2 x = 0 \), \( \forall x \in \mathbb{R} \).

2.) (5 pts) Solve the IVP: \( \frac{dy}{dx} + \frac{1}{2 \sqrt{x-1}} y = \frac{1}{2 \sqrt{x-1}}, y(2) = 0 \).

1.) \( y = e^c \)  
\( \Rightarrow e = (c-2)(c+1) \cos^2 x, \forall x \in \mathbb{R} \)  
\( \Rightarrow c = 2, c = -1 \)  
\( y = 2 \) and \( y = -1 \) are constant solutions.

Now, assume \( y \neq 2, -1 \)

The equation is separable

\[
\left( \frac{1}{y-2} + \frac{1}{y+1} \right) \frac{dy}{dx} = \cos^2 x
\]

Partial fraction:

\[
\frac{a}{y-2} + \frac{b}{y+1} \Rightarrow a = \frac{2}{3}, b = \frac{1}{3}
\]

Thus,

\[
\int \left( \frac{2}{3} + \frac{1}{3} \right) dy = \int \cos^2 \left( \frac{2}{3} \sin x \right) dx
\]

\[
\frac{2}{3} \ln |y-2| + \frac{1}{3} \ln |y+1| = \sin x - \frac{2}{3} \sin x + C
\]

\[
\ln \left( (y-2)^2 (y+1)^2 \right) = 3 \sin x - \sin^2 x + C
\]

\( (y-2)^2 (y+1)^2 = C \in \mathbb{R} \)

Implicit solution

2.) This is a linear DE

\( \frac{dx}{\sqrt{x-1}} = e^{\sqrt{x-1}} \)

We multiply both sides by \( e^{-\sqrt{x-1}} \).

\( \frac{d}{dx} \left( ye^{\sqrt{x-1}} \right) = \frac{1}{2 \sqrt{x-1}} e^{\sqrt{x-1}} \)

We integrate, with respect to \( x \).

\[
y e^{\sqrt{x-1}} = \int \frac{1}{2 \sqrt{x-1}} e^{\sqrt{x-1}} \, dx
\]

\[
= e^{\sqrt{x-1}} + C
\]

\( \Rightarrow y = 1 + Ce^{-\sqrt{x-1}} \)

\( y(2) = 0 \)

\( \Rightarrow 0 = 1 + Ce^{-1}, C = -e \)

\( \Rightarrow y = 1 - e \sin x, x \in (1, \infty) \)