MATH 202.14 (Term 182)

Quiz 2 (Sects. 2.4 & 2.5) Duration: 20min

Name: ID number:

1.)(5pts)Solve the exact DE: $(xe^{x^2} - \cos^2 y + \sin^2 x)\, dx + [x\sin(2y) + \ln y]\, dy = 0$

2.)(5pts)Solve the DE: $\frac{dy}{dx} + \frac{x}{x^2-4} y = \frac{x}{y}$.

1.) $M = xe^{x^2} - \cos^2 y + \sin^2 x$

$N = x\sin(2y) + \ln y$

$M_y = 2\cos y \sin y = \sin(2y)$

$N_x = \sin(2y)$

$\Rightarrow$ $M_y = N_x$ DE is exact

$$\begin{cases} \frac{\partial f}{\partial x} = xe^{x^2} - \cos^2 y + \sin^2 x & (1) \\ \frac{\partial f}{\partial y} = x\sin(2y) + \ln y & (2) \end{cases}$$

$(1) \Rightarrow f(x,y) = \int \left(xe^{x^2} - \cos^2 y + \frac{1-\cos 2x}{2}\right) dx$

$= \boxed{\frac{1}{2}e^{x^2} - x\cos^2 y + \frac{x}{2} - \frac{\sin 2x}{4} + g(y)}$

We substitute $f$ into (2)

$\underbrace{2x\cos y \sin y}_{x\sin 2y} + g'(y) = x\sin 2y + \ln y$

$\Rightarrow g'(y) = \ln y$

$g(y) = \underbrace{\int \ln y\, dy}_{\text{integration by parts}}$

$u = \ln y \quad u' = \frac{1}{y}$

$v' = 1 \quad v = y$

$g(y) = y\ln y - y$

$$\boxed{\frac{1}{2}e^{x^2} - x\cos^2 y + \frac{x}{2} - \frac{\sin 2x}{4} + y\ln y - y = C}$$

$y$ is an implicit solution

2.) This is a Bernoulli's DE, $\alpha = -1$

$u = y^{1-(-1)} = y^2 \Leftrightarrow y = u^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}u^{-\frac{1}{2}}\frac{du}{dx}$

We substitute into the DE.

$\frac{1}{2}u^{-\frac{1}{2}}\frac{du}{dx} + \frac{x}{x^2-4}u^{1/2} = xu^{-1/2}$

$\frac{du}{dx} + \frac{2x}{x^2-4}u = 2x$

This is a linear DE

$e^{\int \frac{2x}{x^2-4}dx} = e^{\ln|x^2-4|} = x^2-4,\ \underline{x>2}$

$\frac{d}{dx}\left(u(x^2-4)\right) = 2x(x^2-4)$

$u(x^2-4) = \int \underbrace{2x(x^2-4)}_{2x^3-8x} dx$

$= \frac{1}{2}x^4 - 4x^2 + C$

$u = \frac{\frac{x^4}{2} - 4x^2 + C}{x^2-4}$

$$\boxed{y^2 = \frac{\frac{x^4}{2} - 4x^2 + C}{x^2-4}},\ x>2$$

$y$ is an implicit solution