MATH 202.14 (Term 182)

Quiz 3 (Sects. 4.1.3, 4.2 & 4.3) Duration: 20min

Name: ID number:

1.)(3pts) Let L be a linear differential operator. If $Ly_{p_1} = e^x$, $Ly_{p_1} = e^{3x}$ and $Ly_{p_2} = e^{-3x}$, then find a particular solution of the DE $Ly = -2e^{2x}\cosh x + 5e^{-x}\sinh(2x)$.

2.)(4pts)Use reduction of order to find a second solution y_2 of the DE: $y'' - y' + g(x)y = 0$, giving that $y_1 = \sqrt{1+e^{2x}}$ is a solution.

3.)(3pts)Solve the DE: $y''' - 3y'' + 7y' - 5y = 0$.

Solution

1.) $-2e^{2x}\cosh x + 5e^{-x}\sinh(2x)$

$= -2e^{2x}\left(\frac{e^{x}+e^{-x}}{2}\right) + 5e^{-x}\left(\frac{e^{2x}-e^{-2x}}{2}\right)$

$= -(e^{3x}+e^{x}) + \frac{5}{2}(e^{x}-e^{-3x})$

$= \frac{3}{2}e^{x} - e^{3x} - \frac{5}{2}e^{-3x}$

$\Rightarrow \boxed{y_p = \frac{3}{2}y_{p_1} - y_{p_2} - \frac{5}{2}y_{p_3}}$

2.) $y'' - y' + g(x)y = 0$

$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2}\,dx$

$P(x) = -1 \Rightarrow e^{-\int P(x)dx} = e^{\int dx} = e^{x}$

$\Rightarrow y_2 = y_1 \int \frac{e^{x}}{1+e^{2x}}\,dx$

substitution $u = e^{x}$, $du = e^{x}dx$

$\int \frac{du}{1+u^2} = \tan^{-1}u = \tan^{-1}(e^{x})$

$\boxed{y_2 = \tan^{-1}(e^{x})\sqrt{1+e^{2x}}}$

3.) Auxiliary equation

$m^3 - 3m^2 + 7m - 5 = 0$

$m = 1$ is a root.

$$\begin{array}{l|l} m^3-3m^2+7m-5 & m-1 \\ \hline -(m^3-m^2) & m^2-2m+5 \\ \quad -2m^2+7m-5 & \\ \quad -(-2m^2+2m) & \\ \qquad 5m-5 & \end{array}$$

$\Delta = 4-20 = -16$

$m_1 = \frac{2-4i}{2}$

$m_2 = \frac{2+4i}{2}$

$m_1 = 1$

$m_2 = 1+2i$

$m_3 = 1-2i$

$\Rightarrow \boxed{y = c_1 e^{x} + c_2 e^{x}\cos 2x + c_3 e^{x}\sin 2x}$