MATH 202.14 (Term 182)

Quiz 4 (Sects. 4.5, 4.6 & 4.7) Duration: 20min

Name: ID number:

1.)(5pts)Solve the DE: $y'' + y' - 2y = \frac{e^x}{e^x + e^{-x}}$.

2.)(5pts)Solve the DE: $x^3y''' + 4x^2y'' + 2xy' - 2y = 0$.

---

1) $y'' + y' - 2y = 0$

The auxiliary equation is

$m^2 + m - 2 = 0, \quad m = 1, -2$

$\Rightarrow y_c = c_1 e^x + c_2 e^{-2x}$

$y_p = u_1 e^x + u_2 e^{-2x}$

$W = \begin{vmatrix} e^x & e^{-2x} \\ e^x & -2e^{-2x} \end{vmatrix} = -3e^{-x}$

$u_1' = -\frac{e^{-2x}}{-3e^{-x}} \frac{e^x}{e^x + e^{-x}} = \frac{1}{3} \frac{e^x}{e^{2x}+1}$

$u_1 = \frac{1}{3}\int \frac{e^x}{e^{2x}+1} dx \overset{\substack{v = e^x \\ dv = e^x dx}}{=\!=} \frac{1}{3}\int \frac{dv}{v^2+1}$

$u_1 = \frac{1}{3}\tan^{-1}(e^x)$

$u_2' = \frac{e^x}{-3e^{-x}} \frac{e^x}{e^x + e^{-x}} = -\frac{1}{3} \frac{e^{4x}}{e^{2x}+1}$

$u_2 = -\frac{1}{3}\int \frac{e^{4x}}{e^{2x}+1} dx \overset{\substack{v = e^x \\ dv = e^x dx}}{=\!=} -\frac{1}{3}\int \frac{v^3 dv}{v^2+1}$

$\frac{v^3}{v^2+1} = v - \frac{1}{2}\frac{2v}{v^2+1}$

$u_2 = -\frac{1}{3}\left[\frac{e^{2x}}{2} - \frac{1}{2}\ln(e^{2x}+1)\right]$

$\Rightarrow y_p = \frac{e^x}{3}\tan^{-1}(e^x) - \frac{e^{-2x}}{3}\left[\frac{e^{2x}}{2} - \frac{1}{2}\ln(e^{2x}+1)\right]$

$y = y_c + y_p$

2.) $y = x^m, \quad x \in (0, \infty)$

The auxiliary equation is

$m(m-1)(m-2) + 4m(m-1) + 2m - 2 = 0$

$(m-1)\left[m(m-2) + 4m + 2\right] = 0$

$(m-1)(m^2 + 2m + 2) = 0$

$m = 1, \quad m^2 + 2m + 2 = 0$

$D = 4 - 8 = -4$

$m = \frac{-2 \pm 2i}{2} = -1 \pm i$

$y = c_1 x + c_2 x^{-1}\cos(\ln x) + c_3 x^{-1}\sin(\ln x)$