MATH 202.14 (Term 182)

Quiz 5 (Sects. 6.2 & 6.3) **Duration: 20min**

Name: **ID number:**

Solve (1 & 2), or (2 & 3)

1.)(7pts)Find 2 powers series solutions of the DE: $(1-x^2)y''-y'-y=0$.

2.)(3pts)Find the indicial roots of the DE $2x^2y''+x(x^6-4)y'+(2-x)y=0$ **at** $x_0=0$.

3.)(7pts)Find a power series solution $y=\sum_{n=0}^{\infty}c_nx^{n+\frac{1}{3}}$ **of the DE** $3xy''+2y'-y=0$.

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1) $y=\sum_{n=0}^{\infty}c_nx^n$, $|x|<1$

$$(1-x^2)\sum_{n=2}^{\infty}c_nn(n-1)x^{n-2}-\sum_{n=1}^{\infty}c_nnx^{n-1}-\sum_{n=0}^{\infty}c_nx^n=0$$

$$\sum_{n=2}^{\infty}n(n-1)c_nx^{n-2}-\sum_{n=2}^{\infty}n(n-1)c_nx^n-\sum_{n=1}^{\infty}nc_nx^{n-1}-\sum_{n=0}^{\infty}c_nx^n=0$$

$$\sum_{k=0}^{\infty}(k+1)(k+2)c_{k+2}x^k-\sum_{k=2}^{\infty}k(k-1)c_kx^k-\sum_{k=0}^{\infty}(k+1)c_{k+1}x^k-\sum_{k=0}^{\infty}c_kx^k=0$$

$$2c_2-c_1-c_0+(6c_3-2c_2-c_1)x+\sum_{k=2}^{\infty}\left[(k+1)(k+2)c_{k+2}-(k+1)c_{k+1}-(k^2-k+1)c_k\right]x^k=0$$

$$\begin{cases} c_2=\frac{1}{2}(c_0+c_1) \\ c_3=\frac{1}{6}(2c_2+c_1)=\frac{1}{6}(c_0+2c_1) \\ c_{k+2}=\frac{1}{(k+1)(k+2)}\left[(k+1)c_{k+1}-(k^2-k+1)c_k\right], k=2,3,\dots \end{cases}$$

- $c_1=0$, $c_0\neq0$

  $c_2=\frac{c_0}{2}$, $c_3=\frac{c_0}{6}$, $c_4=\frac{-1}{12}c_0$, $c_5=\dots$

- $c_0=0$, $c_1\neq0$

  $c_2=\frac{c_1}{2}$, $c_3=\frac{1}{3}c_1$, $c_4=-\frac{1}{24}c_1$, $c_5=\dots$

Thus,

$$y=c_0\underbrace{\left(1+\frac{x^2}{2}+\frac{x^3}{6}-\frac{x^4}{12}+\dots\right)}_{y_1}$$

$$y=c_1\underbrace{\left(x+\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{24}+\dots\right)}_{y_2}$$

$$y=c_1y_1+c_2y_2$$

2.) $p(x)=\frac{x^6-4}{2}\Rightarrow p_0=-2$

$q(x)=\frac{2-x}{2}\Rightarrow q_0=1$

$r(r-1)-2r+1=0$, $r^2-3r+1=0$

$r_1=\frac{3-\sqrt{5}}{2}$, $r_2=\frac{3+\sqrt{5}}{2}$

3.)

$$\sum_{n=0}^{\infty}3c_n\left(n+\tfrac{1}{3}\right)\left(n-\tfrac{2}{3}\right)x^{n-\frac{2}{3}}+\sum_{n=0}^{\infty}2\left(n+\tfrac{1}{3}\right)c_nx^{n-\frac{2}{3}}-\sum_{n=0}^{\infty}c_nx^{n+\frac{1}{3}}=0$$

$$x^{\frac{1}{3}}\left(\sum_{n=0}^{\infty}3c_n\left(n+\tfrac{1}{3}\right)\left(n-\tfrac{2}{3}\right)x^{n-1}+\sum_{n=0}^{\infty}2c_n\left(n+\tfrac{1}{3}\right)x^{n-1}-\sum_{n=0}^{\infty}c_nx^n\right)=0$$

$$\sum_{k=0}^{\infty}\left[c_{k+1}(k+1)(3k+4)-c_k\right]x^k=0$$

$$c_{k+1}=\frac{c_k}{(k+1)(3k+4)}, k=0,1,2,\dots$$

$c_0\neq0$ $\quad c_1=\frac{c_0}{4}$, $c_2=\frac{c_1}{14}=\frac{c_0}{56}$

$c_3=\dots$

$$y=x^{\frac{1}{3}}\left(c_0+c_1x+c_2x^2+c_3x^3+\dots\right)$$

$$=x^{\frac{1}{3}}c_0\left(1+\frac{x}{4}+\frac{x^2}{56}+\dots\right)$$

$$y_1=x^{\frac{1}{3}}\left(1+\frac{x}{4}+\frac{x^2}{56}+\dots\right)$$

is a power series solution