MATH 202.14 (Term 182)

Quiz 6 (Sects. 8.2 & 8.3) **Duration: 20min**

Name: **ID number:**

Solve (1 & 3), or (2 & 3)

1.)(5pts)Solve the IVP $X' = \begin{pmatrix} 6 & -1 \\ 4 & 2 \end{pmatrix} X$, $X(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

2.)(5pts)Solve the IVP $X' = \begin{pmatrix} -2 & -3 \\ 1 & -2 \end{pmatrix} X$, $X(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

3.)(5pts)Solve the system $X' = AX + \begin{pmatrix} 0 \\ t^2 \end{pmatrix}$, **given that** $\Phi(t) = \begin{pmatrix} e^{3t} & -2e^{-2t} \\ e^{3t} & e^{-2t} \end{pmatrix}$ **is a fundamental matrix of** $X' = AX$.

1.) $\begin{vmatrix} 6-\lambda & -1 \\ 4 & 2-\lambda \end{vmatrix} = 0 \Leftrightarrow (\lambda-4)^2 = 0, \ \lambda = 4, 4$

$(A-4I)K = 0$

$\left(\begin{array}{cc|c} 2 & -1 & 0 \\ 4 & -2 & 0 \end{array}\right)$ $\quad 2x - y = 0 \quad K\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$(A-4I)P = K$

$\left(\begin{array}{cc|c} 2 & -1 & 1 \\ 4 & -2 & 2 \end{array}\right)$ $\quad 2x - y = 1 \quad P\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$\Rightarrow X = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{4t} + c_2 \left[t\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right] e^{4t}$

$X(0) = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow \begin{matrix} c_1 = 1 \\ c_2 = 3 \end{matrix}$

$\Rightarrow X = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{4t} + 3 \begin{pmatrix} t \\ 2t-1 \end{pmatrix} e^{4t}, \quad t \in (-\infty, \infty)$

2.) $\begin{vmatrix} -2-\lambda & -3 \\ 1 & -2-\lambda \end{vmatrix} = 0 \Leftrightarrow (2+\lambda)^2 = -3,$ $\lambda = -2 \pm i\sqrt{3}$

$(A - (-2+i\sqrt{3})I)K = 0$

$\left(\begin{array}{cc|c} -\sqrt{3}i & -3 & 0 \\ 1 & -\sqrt{3}i & 0 \end{array}\right) = 0 \quad \begin{matrix} +\sqrt{3}i x + 3y = 0 \\ x = \sqrt{3}i \Rightarrow y = 1 \end{matrix}$

$K \begin{pmatrix} \sqrt{3}i \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + i \begin{pmatrix} \sqrt{3} \\ 0 \end{pmatrix}$

$X_1 = \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos\sqrt{3}t - \begin{pmatrix} \sqrt{3} \\ 0 \end{pmatrix} \sin\sqrt{3}t \right] e^{-2t}$

$X_2 = \left[\begin{pmatrix} \sqrt{3} \\ 0 \end{pmatrix} \cos\sqrt{3}t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin\sqrt{3}t \right] e^{-2t}$

$X( ) = c_1 X_1 + c_2 X_2$

$X(0) = c_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} \sqrt{3} \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} c_1 = 1 \\ c_2 = -\frac{\sqrt{3}}{3} \end{matrix}$

3) $\Phi^{-1} = \frac{1}{3e^{t}} \begin{pmatrix} e^{-2t} & 2e^{-2t} \\ -e^{3t} & e^{3t} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} e^{-3t} & 2e^{-3t} \\ -e^{2t} & e^{2t} \end{pmatrix}$

$\Phi^{-1}F = \frac{1}{3} \begin{pmatrix} 2t^2 e^{-3t} \\ t^2 e^{2t} \end{pmatrix}$

$\int \Phi^{-1}F = \frac{1}{3} \begin{pmatrix} 2\left(\frac{t^2}{-3} - \frac{2t}{9} + \frac{2}{-27}\right) e^{-3t} \\ \left(\frac{t^2}{2} - \frac{2t}{4} + \frac{4}{8}\right) e^{2t} \end{pmatrix}$

$\int t^2 e^{at} = \left(\frac{t^2}{a} - \frac{2t}{a^2} + \frac{2}{a^3} \right) e^{at}$

$X_p = \Phi \int \Phi^{-1} F$

$= \frac{1}{3} \begin{pmatrix} -\frac{5}{3}t^2 + \frac{5}{9}t + \frac{23}{27} \\ -\frac{t^2}{6} - \frac{17}{18}t + \frac{19}{54} \end{pmatrix}$

$X = \Phi C + X_p$