Quiz 1
Math 371, Introduction to Numerical Computing
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Question 1. [1 mark] In high school, some students have been misled to believe that 22/7 is either the actual value of $\pi$ or an acceptable approximation to it. Show that 355/113 is a better approximation in terms of both absolute and relative errors.

Question 2. [1 mark] Consider the functions $f(x) = \sqrt[3]{4x-1}$ and $g(x) = \sin x$.

i [ 1/2 mark ] Find the linear Taylor polynomial of $f$ about 4.25.

ii [ 1/2 mark ] Find the error upper bound in approximating $g$ by $x - x^3/6$.

Question 3. [2 marks] The Maclaurin series for $(1 + x)^n$ is also known as the binomial series. It states that

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \quad \text{(valid for } |x| < 1.) \quad (1)$$

i [ 1/2 mark ] Derive this series.

ii [ 1/2 mark ] Give its particular form in summation notation ($\Sigma$) for $n = 1/2$.

iii [ 1/2 mark ] Use the first four terms in the last form to compute $\sqrt{1.0001}$ rounded to five significant places.

iv [ 1/2 mark ] Use Series (1) to obtain Maclaurin Series for $1/(1 - x)$.

Question 4. [2 marks] Consider the floating-point system $\mathbb{F}(10, 4, -100, 100)$.

i [ 1/2 mark ] Determine the machine representation for the following numbers:
(a) $10^{-30}$. (b) $64.015625$. (c) $-8 \times 10^{-51}$. (d) $-8 \times 100^{-51}$.

ii [ 1/2 mark ] Which of these are machine numbers?
(a) $10^{103}$. (b) $2 \times 10^{-32}$. (c) $\frac{1}{5}$. (d) $-\frac{1}{7}$.

iii [ 1/2 mark ] What are the machine numbers immediately to the right and left of $10^5$? Also, in the case of machine underflow, what is the relative error involved in replacing a nonzero number $x$ by zero?

iv [ 1/2 mark ] Determine the chopping value of the rational number 335/113.