1. Find the Laurent series for the following functions in given domains.
(a) \(f(z) = (z + 1)/(z - 1)\) on \(\{z : |z| > 1\}\).
(b) \(g(z) = z^2(1 - z)^{-1}\) on \(\{z : 0 < |z| < 1\}\).
2. Let $f$ be an analytic function on the closed unit disc $\overline{D} = \{ z \in \mathbb{C} : |z| \leq 1 \}$. Suppose that $f(0) = 0$.

(a) Prove that $0$ is a removable singularity of the function $g(z) := f(z)/z$ and as a consequence,

$$\tilde{g}(z) := \begin{cases} \frac{f(z)}{z} & \text{if } z \neq 0 \\ f'(0) & \text{if } z = 0 \end{cases}$$

is analytic on $\overline{D}$.

(b) Assume further that $|f(z)| \leq 1$ on the boundary of the disc $\{ z \in \mathbb{C} : |z| = 1 \}$. Prove $|f'(0)| \leq 1$ (Hint. Apply the maximum modulus principle to the function $\tilde{g}$.)
3. Classify (with explanation) the isolated singularities of the following functions.

(a) \( f(z) = \sin \frac{1}{z^2} \).
(b) \( g(z) = \frac{z^2}{\sin^3 z} \).
(c) \( h(z) = \cot z = \frac{\cos z}{\sin z} \).
4. Compute the following residues.
   (a) $\text{Res} \left( (2z^2 - 3z + 1)e^{1/z}; 0 \right)$.
   (b) $\text{Res} \left( z \cot^2 z; 0 \right)$.
5. Compute
\[ \int_0^{2\pi} \frac{d\theta}{5 + 4\sin \theta}. \]
6. Compute

\[ \int_0^\infty \frac{x^3 \sin 2x}{x^4 + 1} \, dx. \]
7. Compute
\[ \int_0^\infty \frac{\sqrt{x}}{x^2 + 4} \, dx. \]
8. Let $P$ be a degree $n$ polynomial and let $a_1, \ldots, a_n$ be zeros of $P$. That is, $P(z) = C(z - a_1)(z - a_2)\cdots(z - a_n)$ for some constant $C$. Suppose that all $a_j$'s are inside $\gamma$, where $\gamma$ is the positively oriented circle of radius $r > 0$ centered at the origin. Prove that

$$\int_{\gamma} \frac{zP'(z)}{P(z)} \, dz = 2\pi i(a_1 + a_2 + \cdots + a_n).$$