STAT 301: Introduction to Probability Theory  
Semester 182, Second Major Exam  
Tuesday March 19, 2019 (6:30 pm)

Name:  
ID #:  

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<tr>
<th>Question No</th>
<th>Full Marks</th>
<th>Marks Obtained</th>
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**Instructions:**

1. Mobiles are not allowed in exam. If you have your mobile with you, turn it off and put it under your seat so that it is visible to proctor.

2. Make sure you have 8 unique pages of exam paper (including this title page).

3. Show all the calculation steps. There are points for the steps so if you miss them, you would lose points.
Q.No.1: - (10 pts.) Three German Shepherds and four Siberian Huskies enter a dog show. At the end of the show a ranking of 6 dogs is provided. Assume that all rankings are equally likely. Let $X$ denotes the highest ranking obtained by a German Shepherd. For example, $X = 1$ if a German Shepherd is the show winner.

a) What values can $X$ take?

b) Write down the probability mass function of $X$.

c) Write down the cumulative distribution function of $X$.

d) What is $E[X]$?
Q.No.2: - (10 pts.)

a) If $X$ is uniformly distributed over $(0,1)$, find the mean of $Y$ where $Y = e^X$. 
b) A continuous random variable $Z$ has the following probability density function:

$$f(z) = \frac{1}{z\omega \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln z - \theta}{\omega}\right)^2} \quad \text{where} \quad z > 0$$

Derive the mode of $Z$. 
Q.No.3: - (10 pts.)

a) Suppose that a discrete random variable $Y$ has the following probability mass function:

$$f(y) = \frac{(y - 1)}{(\alpha - 1)} \beta^\alpha (1 - \beta)^{y - \alpha}$$

where $y = \alpha, \alpha + 1, \alpha + 2, \ldots$

Derive the moment generating function of $Y$. 

b) A discrete random variable $X$ has the following probability mass function:

$$f(x) = \binom{m}{x} \binom{N-m}{n-x} / \binom{N}{n}; \quad x = \max\{0, n + m - N\} \text{ to } \min\{m, n\}$$

Derive $E[X(X - 1)]$. 
Q.No.4: (10 pts.)

a) Suppose that a help session for a course has a capacity of 650 students, but that invitations are sent out to 4600 students. If each student who receives an invitation has a probability of 0.13 of attending the help session, independently of everybody else, what is the probability that the number of students attending the help session will exceed the capacity?

b) Suppose a certain mechanical component produced by a company has a width that is normally distributed with a mean 2600 and a variance 0.16. If the company needs to be able to guarantee to its purchaser that no more than 1 in 500 of the components have a width less than 2599, by how much does the value of \( \sigma \) need to be reduced?
Q.No.5: (10 pts.)

a) An urn contains 4 white and 4 black balls. We randomly choose 4 balls. If 2 of them are white and 2 are black, we stop. If not, we replace the balls in the urn and again randomly select 4 balls. This continues until exactly 2 of the 4 chosen are white. What is the probability that we shall make more than two selections?

b) Suppose that the Poisson distribution models the number of cars abandoned daily on a certain highway and the probability that there will be no abandoned cars in a day is 0.719. Find the probability that there will be at least 3 abandoned cars in the next week.