1. If $f(x) = x + 72^x$, then $f'(1) =$

(a) $1 + 72 \ln 72$

(b) $1 + 8 \ln 8 + 9 \ln 9$

(c) $1 + 9 \ln 8 + 8 \ln 9$

(d) $1 + \frac{8}{9} \ln 72$

(e) $1 + \frac{9}{8} \ln 72$

2. If $f(x) = \dfrac{x^3}{g(x)}$, $g(-1) = 2$ and $g'(-1) = -9$, then $f'(-1) =$

(a) $\dfrac{-3}{4}$

(b) $\dfrac{5}{4}$

(c) $\dfrac{81}{3}$

(d) $\dfrac{-3}{81}$

(e) $\dfrac{1}{3}$

If $f(x) = \dfrac{x^2}{g(x)}$, $g(-1)=2$ and $g'(-1)=-9$,

then $f'(-1) =$

$\boxed{\dfrac{5}{4}}$

3. $\lim_{h\to 0} \dfrac{\sin\left(\dfrac{\pi}{3}+h\right)\cos\left(\dfrac{\pi}{3}+h\right)-\dfrac{\sqrt{3}}{4}}{h} =$

(a) $-\dfrac{1}{2}$

(b) $\dfrac{1}{2}$

(c) $\sqrt{2}$

(d) $-\sqrt{2}$

(e) $0$

$\lim_{h\to 0} \dfrac{\sin\left(\frac{\pi}{6}+h\right)\cos\left(\frac{\pi}{6}+h\right)-\frac{\sqrt{3}}{4}}{h} =$

$\boxed{+\frac{1}{2}}$

4. $\lim_{x\to 0} \dfrac{\sin(3x)\tan(5x)}{3x^2-5x^4} =$

(a) 5

(b) 3

(c) $-3$

(d) 0

(e) 4

$\lim_{x\to 0} \dfrac{\sin 3x \tan(4x)}{3x^2-5x^4} = 4$

5. A water tank has the shape of an inverted circular cone with base radius $3m$ and height $6m$. If water is being pumped into the tank at a rate of $3\,m^3/min$, find the rate at which the water level is rising when the water is $4m$ deep.
(Volume of the cone is $V = \frac{1}{3}\pi r^2 h$)

(a) $\frac{3}{4\pi}$
(b) $\frac{2}{\pi}$
(c) $\frac{4\pi}{3}$
(d) $\frac{\pi}{2}$
(e) $\frac{3}{\pi}$

6. If $y = 6 - 2u^2$ and $u = \cot\left(\frac{x}{2}\right)$, then $\frac{dy}{dx}$ when $u = \sqrt{3}$ equals to

(a) $8\sqrt{3}$
(b) $4\sqrt{3}$
(c) $10\sqrt{3}$
(d) $6\sqrt{3}$
(e) $\sqrt{3}$

7. Using table of values

| $x$ | 1 | 4 | 6 |
|---|---|---|---|
| $f(x)$ | 4 | 0 | 6 |
| $f'(x)$ | 5 | 7 | 4 |
| $g(x)$ | 4 | 1 | 6 |
| $g'(x)$ | 5 | 8 | 3 |

$$\frac{d}{dx}f(2x+g(x))\Big|_{x=1} + \frac{d}{dx}g(\sqrt{x})\Big|_{x=16} =$$

(a) 29
(b) 28
(c) 30
(d) 27
(e) 16

$$\frac{d}{dx}f(2x+g(x))\Big|_{x=1} + \frac{d}{dx}g(\sqrt{x})\Big|_{x=36}$$

$= 30$

8. If $f(x) = e^{3x}\log_2(2x+1)$, then the slope of the normal line to $f(x)$ at $x = 0$ is

(a) $-\dfrac{\ln 2}{2}$
(b) $-\dfrac{\ln 3}{2}$
(c) $-\dfrac{3\ln 2}{2}$
(d) $\dfrac{3\ln 3}{2}$
(e) $-\dfrac{1}{\ln 2}$

If $f(x) = e^{2x}\log_3(3x+1)$,
then - - - - -
is $\boxed{-\dfrac{\ln 3}{3}}$

9. If $f(x) = 2\sin^{-1}(3x)$, then $(f^{-1})'\left(\frac{\pi}{2}\right) =$

(a) $\frac{1}{6\sqrt{2}}$

(b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{4\sqrt{2}}$

(d) $1$

(e) $\frac{1}{\sqrt{2}}$

10. A particle moves according to a law of motion $s = f(t) = \tan^{-1}(t^2 - 2t + 1), t \geq 0$, where $t$ is measured in seconds and $s$ in meters. The total distance in meters traveled by the particle during the first 2 seconds is

(a) $\frac{\pi}{2}$

(b) $\pi$

(c) $\frac{3\pi}{2}$

(d) $2\pi$

(e) $\frac{\pi}{4}$

11. If $2\sqrt{y} = x - y$, then $\dfrac{d^2y}{dx^2} =$

(a) $\dfrac{1}{2(1+\sqrt{y})^3}$

(b) $\dfrac{x}{(1+\sqrt{y})^2}$

(c) $\dfrac{2x}{(1+\sqrt{y})^3}$

(d) $\dfrac{1}{(1+\sqrt{y})^4}$

(e) $\dfrac{1}{2(1-\sqrt{y})^3}$

If $2\sqrt{y} = x + y$, then $\dfrac{d^2y}{dx^2} =$

$$\boxed{\dfrac{1}{2(1-\sqrt{y})^3}}$$

12. The tangent line to the curve $y = (\sin x)^{x^2}$ at $x = \dfrac{\pi}{2}$ is

(a) $y = 1$

(b) $y = \dfrac{1}{4}(-4 - \pi^2 + 2\pi\, x)$

(c) $y = \dfrac{1}{2}(-2 - \pi^2 + 2\pi\, x)$

(d) $y = \dfrac{1}{2}(-2 - \pi + 2x)$

(e) $y = \dfrac{1}{4}(-4 + \pi^2 - 2\pi\, x)$

13. Let $x$ and $y$ be differentiable functions of $t$ and $\frac{dx}{dt} \neq 0$. If $x^2+y^2-2x=-1$, then for how many points $\frac{dx}{dt} = 2\frac{dy}{dt}$?

(a) one point
(b) two points
(c) three points
(d) no point
(e) four points

14. If $f(x) = \sin x + \cos x$, then $f^{(2019)}(\pi) - f^{(2020)}(\pi) =$

(a) 2
(b) 0
(c) 3
(d) 1
(e) $-1$

15. If the line $y = 3x$ is tangent to the curve $y = x^2 + k$ for some constant $k$, then $\frac{8}{3}k =$

(a) 6
(b) 3
(c) 10
(d) 2
(e) 4

If the line $y = 3x$ is tangent to the curve $y = x^2 + 2k$ for some constant $k$, then $\frac{8}{3}k = \boxed{3}$