1. The domain of $f(x,y) = \ln(9 - x^2 - 9y^2)$ is the set of all points in $R^2$ such that

(a) $\left\{(x,y) : \frac{1}{9}x^2 + y^2 < 1\right\}$

(b) $\left\{(x,y) : x^2 + \frac{1}{9}y^2 < 1\right\}$

(c) $\left\{(x,y) : x^2 + y^2 \leq 3\right\}$

(d) $\{(x,y) : x = \pm 3y\}$

(e) $\{(x,y) : x \in (-\infty, \infty), y \in (-\infty, \infty)\}$

2. An equation of the tangent plane to the surface at $z = xe^{xy}$ at the point $(2,0,2)$ is

(a) $z = x + 4y$

(b) $z = x - 4y$

(c) $z = x + 4y - 4$

(d) $z = x + 4y + 4$

(e) $z = 4x + y$

3. For the function $f(x, y) = x^2 + 4y^2 + 1$, consider the following statements:
(I) The domain of $f$ is $R^2$.
(II) The range of $f$ is $(-\infty, 1)$.
(III) The graph of $f$ is an elliptic paraboloid with vertex $(0, 0, 1)$.

Which of the above statements is (are) **TRUE**:

(a) I and III
(b) I and II
(c) I only
(d) II and III
(e) all of them

4. If $f(x, y, z) = \sqrt{\sin^2 x + \sin^2 y + \sin^2 z}$, then the value of $f_z\left(0, 0, \frac{\pi}{4}\right)$ is

(a) $\frac{1}{\sqrt{2}}$
(b) $\frac{-1}{\sqrt{2}}$
(c) $\sqrt{2}$
(d) $2$
(e) $\frac{1}{2}$

5. Consider the function

$$f(x,y) = \begin{cases} \dfrac{xy^2 \cos y}{x^2 + y^4} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Which of the following statements is **TRUE**?

(a) $f$ is discontinuous at $(0,0)$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = 0$

(c) $f(x,y) \to 2$ as $(x,y) \to (0,0)$ along the curve $x = y^2$

(d) $f(x,y) \to 1$ as $(x,y) \to (0,0)$ along the line $y = x$

(e) the function $f$ can be re-defined at $(0,0)$ to become continuous at $(0,0)$

6. Let $f$ be a differentiable function and $z = f(x^n\, y^n), n \in N = 1, 2, 3, \ldots,$ then $x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} =$

(a) $0$

(b) $x^n - y^n$

(c) $(x^n - y^n)f'(xy)$

(d) $f'(x^n y^n)$

(e) $n(x-y)f'(xy)$

7. If the linearization of the function $f(x,y) = 4\tan^{-1}(xy)$ at the point $(1,1)$ is given by $L(x,y) = 2x + 2y + k$, then $k =$

(a) $\pi - 4$

(b) $0$

(c) $2\pi$

(d) $\pi + 4$

(e) $4 - \pi$

8. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + \sin^2 y}{4x^2 + y^2} =$

(a) doesn't exist

(b) $1$

(c) $\frac{1}{4}$

(d) $0$

(e) $-1$

9. The directional derivative of $f(x, y) = x^3 - 3xy + 4y^2$ at the point $(1, 2)$ in the direction of the unit vector given by angle $\theta = \dfrac{\pi}{3}$ equals

(a) $\dfrac{13\sqrt{3} - 3}{2}$

(b) $\dfrac{13\sqrt{3} + 3}{2}$

(c) $\dfrac{13 - 3\sqrt{3}}{2}$

(d) $\dfrac{13 + 3\sqrt{3}}{2}$

(e) $\dfrac{16\sqrt{3}}{2}$

10. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + 5y^2}} =$

(a) $0$

(b) $1$

(c) $\dfrac{1}{5}$

(d) $-\dfrac{1}{5}$

(e) doesn't exist

11. Let $f(x, y, z) = x^2 - y^2 - z^2$. Consider the following statements:
The level surface(s) of the function $f$ is (are)
(I) a family of concentric spheres.
(II) a family of hyperboloids of two sheets with axis the $x$-axis.
(III) a family of hyperboloids of one sheet with axis the $x$-axis.
(IV) a right circular cone with vertix the origin and axis the $x$-axis.

Which of the above statements is (are) **TRUE**?

(a) II,III and IV only
(b) II and III only
(c) III only
(d) II only
(e) I,II and III only

12. Suppose that the equation $F(x, y, z) = 0$ implicity defines each of the three variables $x, y$, and $z$ as functions of the other two:

$$z = f(x, y),\ \ y = g(x, z),\ \ x = h(y, z).$$

If $F$ differentiable and $F_x$, $F_y$ and $F_z$ are all nonzero, then $\dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial y}\dfrac{\partial y}{\partial z} =$

(a) $-1$
(b) $1$
(c) $0$
(d) $xyz$
(e) varies from one function to another

13. Let $z = e^r \cos\theta$, $r = st$, $\theta = \sqrt{s^2+t^2}$. Then $\dfrac{\partial z}{\partial s} =$

(a) $zt - \dfrac{zs}{\theta}\tan\theta$

(b) $zt - \dfrac{zs}{\theta}\sin\theta$

(c) $e^r\left(t\sin\theta - \dfrac{s}{\theta}\cos\theta\right)$

(d) $e^r\sin\theta\left(t\cot\theta + \dfrac{s}{\theta}\right)$

(e) $zs - \dfrac{zt}{\theta}\tan\theta$

14. The slope of the tangent line to the curve of intersection of paraboloid $z = 4 - x^2 - 2y^2$ and the plane $x = 1$ at the point $(1,1,1)$ is

(a) $-4$

(b) $-2$

(c) $2$

(d) $4$

(e) $-6$

15. The line that passes through $(1, 2, 3)$ and perpendicular to the plane $x + 3y + z = 5$ is also passing through the point

(a) $(0, -1, 2)$
(b) $(2, 5, 0)$
(c) $(3, 0, 5)$
(d) $(-1, 1, 2)$
(e) $(-2, -7, 1)$

16. Consider the following statements about the surface $4x = x^2 - y^2 + z^2 - 2z$.
(I) Its graph has two sheets.
(II) Its vertical traces in any plane $y = k$ are circles.
(III) Its axis is parallel to the $x$-axis.

Which of the above statements is (are) **TRUE**?

(a) II only
(b) I and II only
(c) I and III only
(d) III only
(e) none of them

17. If the minimum rate of change of the function $f(x, y) = 4y\sqrt{x}$ at the point $(k, 1)$ is $-\sqrt{65}$, then one of the possible value of $k$ is

(a) $\frac{1}{16}$

(b) $\frac{1}{4}$

(c) $-4$

(d) $-\frac{1}{16}$

(e) $\frac{1}{8}$

18. If $f(x, y, z) = \sqrt{1 + xz} + \sqrt{1 - xy}$, then $f_{xyz} =$

(a) $0$

(b) $\frac{1}{2}\left[\frac{z}{\sqrt{1+xz}} - \frac{y}{\sqrt{1-xy}}\right]$

(c) $\frac{-1}{4\sqrt{1-xy}}\left[\frac{xy}{1-xy} + 2\right]$

(d) $\frac{x}{2\sqrt{1+xz}}$

(e) $\frac{-x}{2\sqrt{1-xy}}$

19. Let $a^2 + b^2 + c^2 = 1$. Which one of the following statements is **FALSE**?

(a) the distance $D$ between the parallel planes
$ax + by + cz + d_1 = 0$, $ax + by + cz + d_2 = 0$ is $D = |d_1 + d_2|$.

(b) every plane has exactly two unit normal vectors.

(c) if a plane is parallel to one of the coordinate planes, then its normal vector is parallel to one of the three vectors $\vec{i}$, $\vec{j}$, or $\vec{k}$.

(d) if two planes intersect in a line $L$, then $L$ is parallel to the cross product of the normals to the two planes.

(e) the distance from the point $p(x_0, y_0, z_0)$ to the plane
$ax+by+cz = 0$ is the absolute value of the dot product of the vectors $< a, b, c >$ and $< x_0, y_0, z_0 >$; that is $| < a, b, c > \cdot < x_0, y_0, z_0 > |$.

20. Let $L_1$ and $L_2$ be the lines whose parametric equations are

$$L_1 : x = 1 + 2t, \quad y = 2 - t, \quad z = 4 - 2t$$

$$L_2 : x = 9 + t, \quad y = 5 + 3t, \quad z = -4 - t.$$

The parametric equations for the line that is perpendicular to $L_1$ and $L_2$ and passes through their point of intersection are:

(a) $x = 7 + t, \quad y = -1, \quad z = -2 + t$

(b) $x = 7 + 7t, \quad y = -1, \quad z = -2 + t$

(c) $x = -7 - 7t, \quad y = 1, \quad z = 2 - t$

(d) $x = -7 + t, \quad y = 1, \quad z = 2 + t$

(e) $x = 7 + t, \quad y = -1, \quad z = -2 + 7t$